10-703 Recitation 4

Topics (Lectures 1-8):

1. Introduction to Reinforcement and Representation Learning
2. Multi-Arm Bandits
3. MDPs, Value and Policy Iteration
4. Monte Carlo Learning, Temporal Difference Learning, Monte Carlo Tree Search
5. Function Approximation, Deep Q learning
6. Policy gradients, REINFORCE, Actor-Critic methods

*****Note this is not an exhaustive list. Anything covered in lectures in fair game.**

What to expect?

- Open note
  - Can only use downloaded content
  - No internet use
- Types of questions:
  - True/False (with and without explanation)
  - Select all that apply (if explain - **explain each choice why you did or did not select it**)
  - Short answer
  - **If there is a box to explain, always explain your answer. Otherwise you will not get full credit.

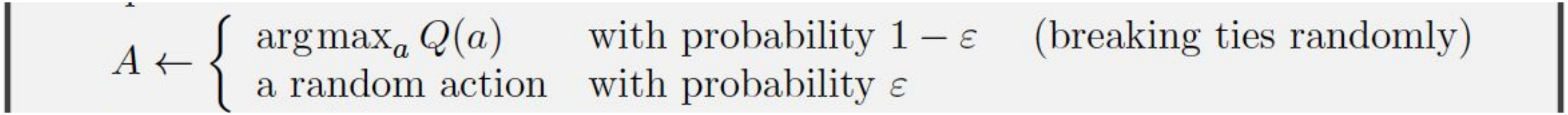
Bandits

- You have one state with k actions
- Each action gets you a reward
- Want to maximize reward in least amount of time
  - How to sample actions to do this efficiently?

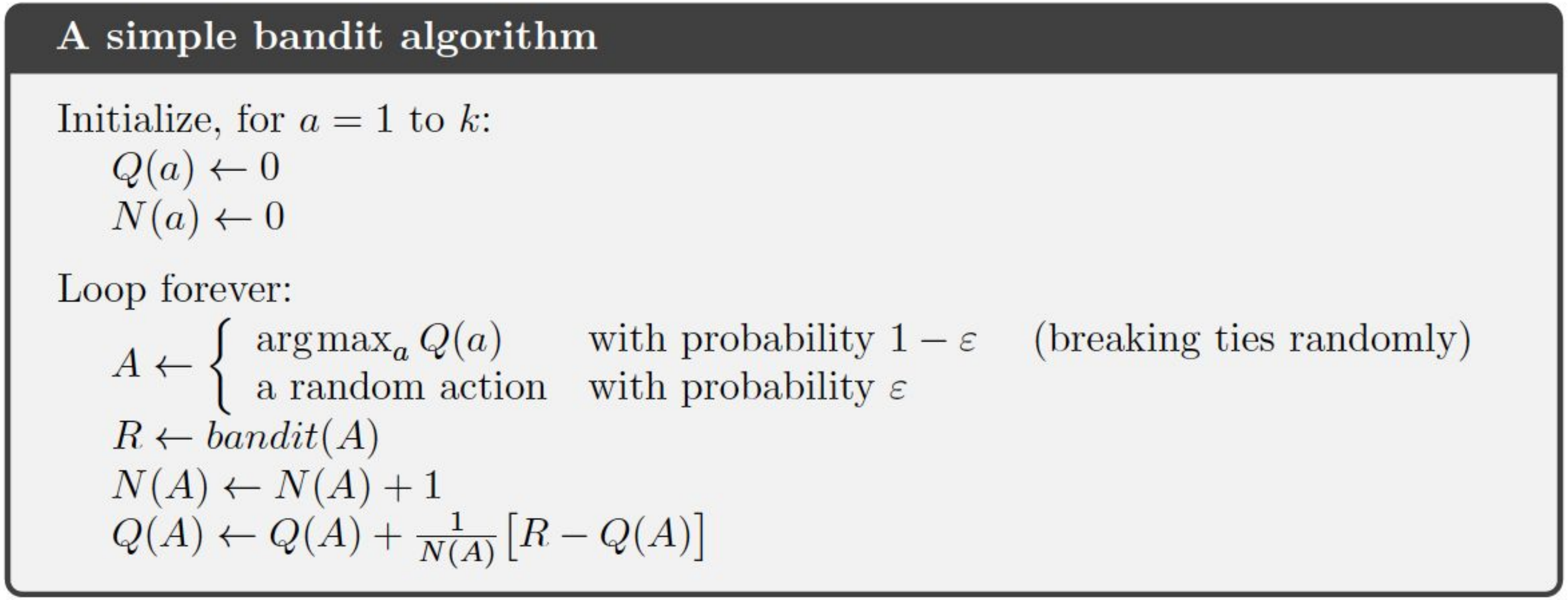
$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}},$$

Greedy action: $$A_t \doteq \underset{a}{\arg\max}\, Q_t(a),$$

Epsilon-greedy bandits



$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$



A simple bandit algorithm

```
Initialize, for $a = 1$ to $k$:
    $Q(a) \leftarrow 0$
    $N(a) \leftarrow 0$

Loop forever:
    $A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$
    $R \leftarrow bandit(A)$
    $N(A) \leftarrow N(A) + 1$
    $Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\big[R - Q(A)\big]$
```

Upper confidence bound

- the square-root term is a measure of the uncertainty or variance in the estimate of a’s value
- As Nt(a) increases the uncertainty term decreases.
- On the other hand, each time an action other than a is selected, t increases but Nt(a) does not, causing the uncertainty to increase
- The use of the natural logarithm means that the increases get smaller over time, but are unbounded

$$A_t \doteq \underset{a}{\arg\max}\left[Q_t(a) + c\sqrt{\frac{\ln t}{N_t(a)}}\right],$$

Optimistic Initial Values

- Set initial Q values much higher than the reward
- Encourage some exploration initially
- Whichever actions are initially selected, the reward is less than the starting estimates; the learner switches to other actions, being “disappointed” with the rewards it is receiving. The result is that all actions are tried several times before the value estimates converge. The system does a fair amount of exploration even if greedy actions are selected all the time.

Gradient Bandit Algorithms

$$\Pr\{A_t = a\} \doteq \frac{e^{H_t(a)}}{\sum_{b=1}^{k} e^{H_t(b)}} \doteq \pi_t(a),$$

$$H_{t+1}(A_t) \doteq H_t(A_t) + \alpha\big(R_t - \bar{R}_t\big)\big(1 - \pi_t(A_t)\big), \qquad \text{and}$$
$$H_{t+1}(a) \doteq H_t(a) - \alpha\big(R_t - \bar{R}_t\big)\pi_t(a), \qquad \text{for all } a \neq A_t,$$

$\bar{R}_t \in \mathbb{R}$ is the average of all the rewards

MDPs

A **Finite** Markov Decision Process is a tuple $(\mathcal{S}, \mathcal{A}, T, r, \gamma)$

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- p is one step dynamics function
- r is a reward function
- γ is a discount factor $\gamma \in [0,1]$

policy: $\pi(a|s)$

* denotes optimal $q_*(s,a) \doteq \max_\pi q_\pi(s,a),$

State-value function:

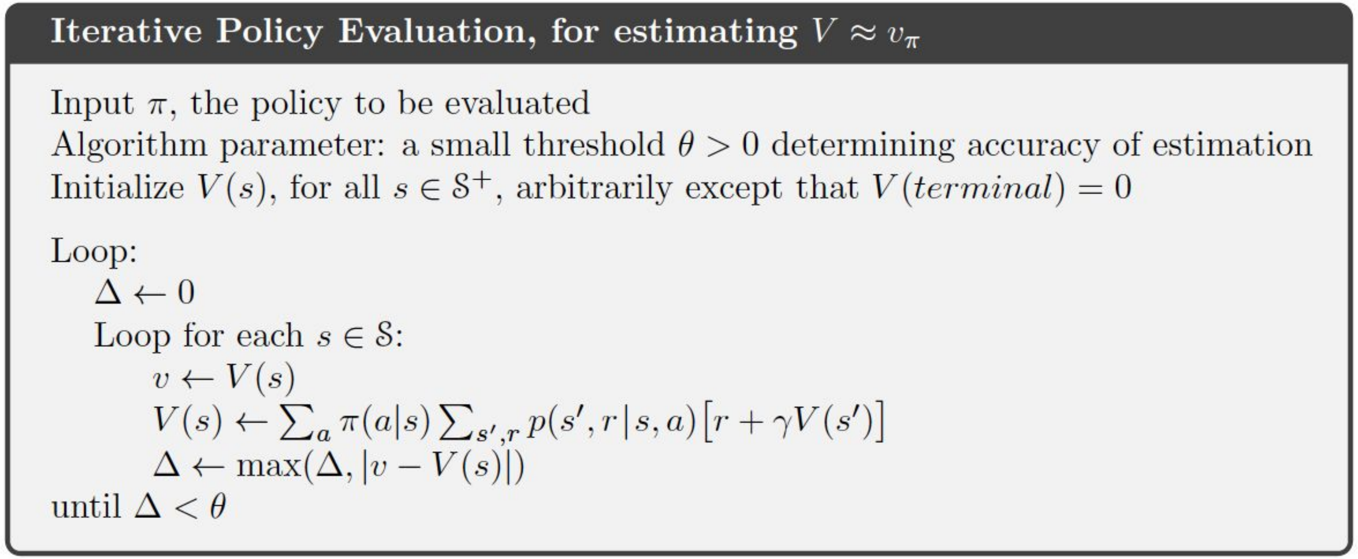
- $v_\pi(s) = \mathbb{E}[G_t \mid S_t = s]$

Action-value function:

- $q_\pi(s,a) = \mathbb{E}[G_t \mid S_t = s, A_t = a]$

Policy evaluation

- Find value function for a given policy
- Converges to unique true value function in limit
- In practice, use iterative policy evaluation (below) - stop when max delta below a threshold
- Can update value function “in place” or use two copies



Iterative Policy Evaluation, for estimating $V \approx v_\pi$

```
Input $\pi$, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
    $\Delta \leftarrow 0$
    Loop for each $s \in \mathcal{S}$:
        $v \leftarrow V(s)$
        $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
        $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$
```

Bellman Backup & Contraction Mapping Theorem

- value function v_pi is the unique solution to its Bellman equation.
- Bellman backup operator is gamma-contraction

$$\begin{aligned} v_{k+1}(s) &\doteq \mathbb{E}_\pi[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)\Big[r + \gamma v_k(s')\Big], \end{aligned}$$

- Define the Bellman expectation backup operator

$$F^\pi(\mathrm{v}) = r^\pi + \gamma T^\pi \mathrm{v}$$

Contraction Mapping Theorem

An operator F on a normed vector space $\mathcal{X}$ is a γ-**contraction**, for $0 < \gamma < 1$ provided for all $x, y \in \mathcal{X}$:

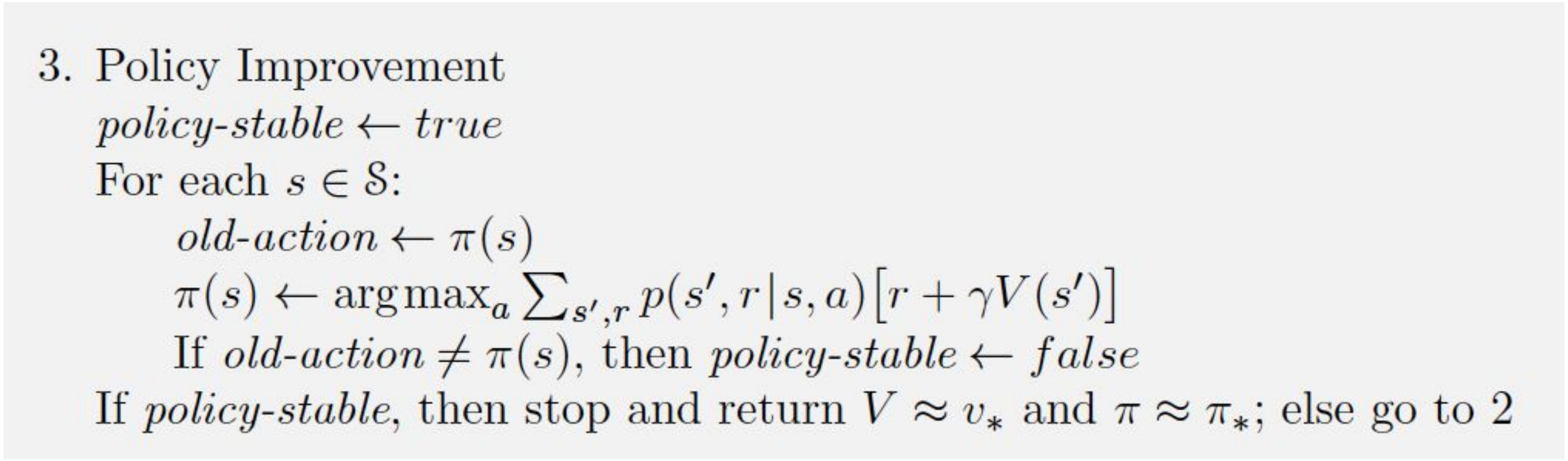
$$\|F(x) - F(y)\| \leq \gamma \|x - y\|$$

Theorem (Contraction mapping)
For a γ-contraction F in a complete normed vector space $\mathcal{X}$:

- F converges to a unique fixed point in $\mathcal{X}$,
- at a linear convergence rate γ.

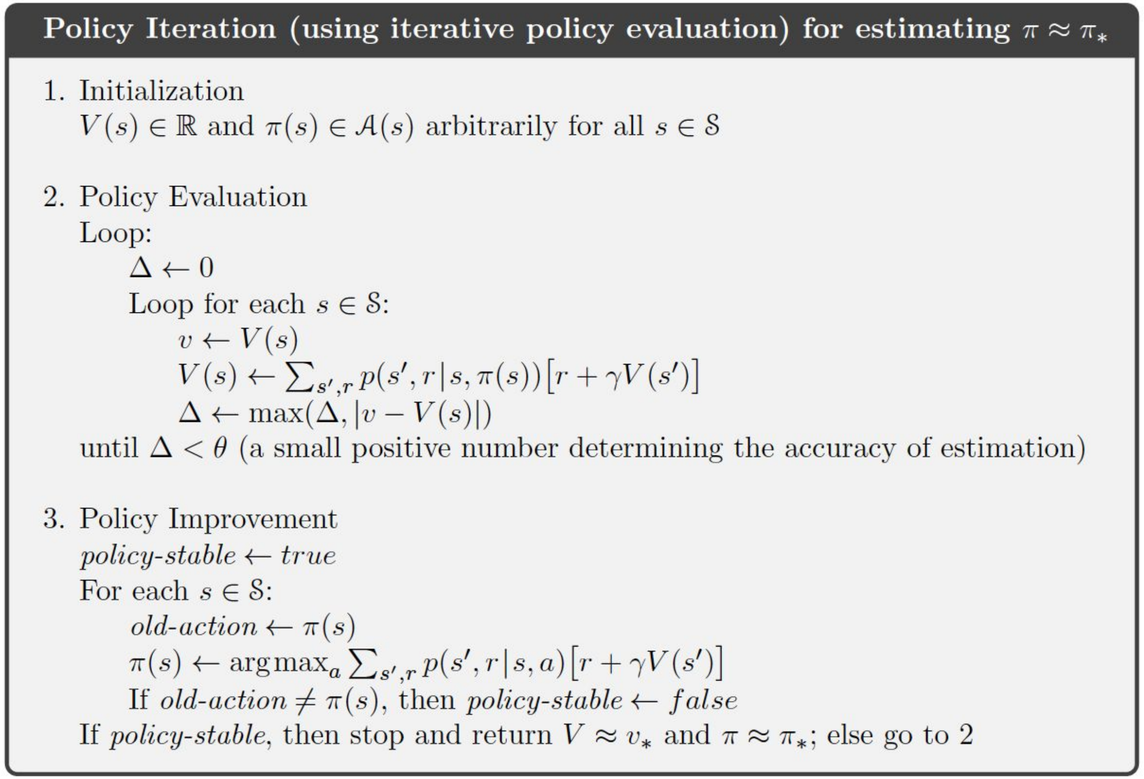
Policy improvement

- Given value function for current policy, do one-step look-ahead and check if it is better to change policy to new action in each state
- Strictly improving except when policy is already optimal



```
3. Policy Improvement
   policy-stable ← true
   For each $s \in \mathcal{S}$:
       old-action ← $\pi(s)$
       $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s', r|s, a)\big[r + \gamma V(s')\big]$
       If old-action $\neq \pi(s)$, then policy-stable ← false
   If policy-stable, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2
```

Policy iteration

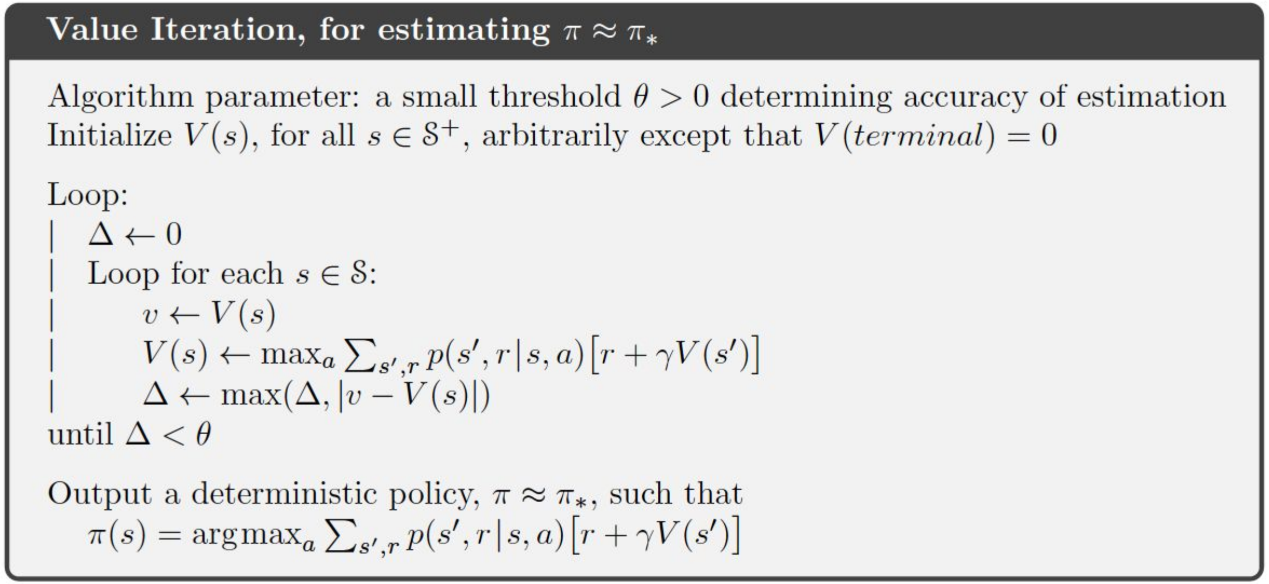


Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

```
1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Loop:
      $\Delta \leftarrow 0$
      Loop for each $s \in \mathcal{S}$:
         $v \leftarrow V(s)$
         $V(s) \leftarrow \sum_{s',r} p(s', r \mid s, \pi(s))\big[r + \gamma V(s')\big]$
         $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   $policy\text{-}stable \leftarrow true$
   For each $s \in \mathcal{S}$:
      $old\text{-}action \leftarrow \pi(s)$
      $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma V(s')\big]$
      If $old\text{-}action \neq \pi(s)$, then $policy\text{-}stable \leftarrow false$
   If $policy\text{-}stable$, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2
```

Value iteration

- Combine policy evaluation and policy iteration in each state sweep
- Effectively combines one sweep of policy evaluation and one sweep of policy improvement.
- Often much faster convergence than policy iteration

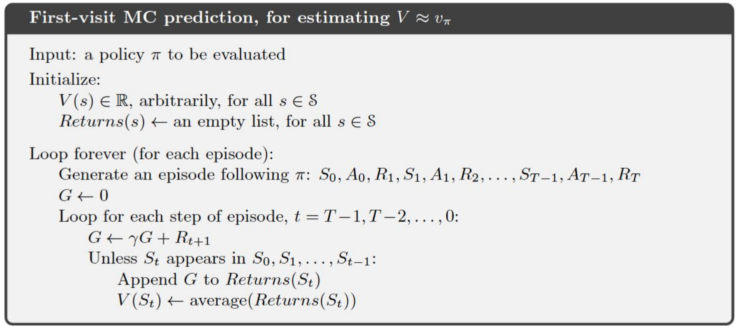


Value Iteration, for estimating $\pi \approx \pi_*$

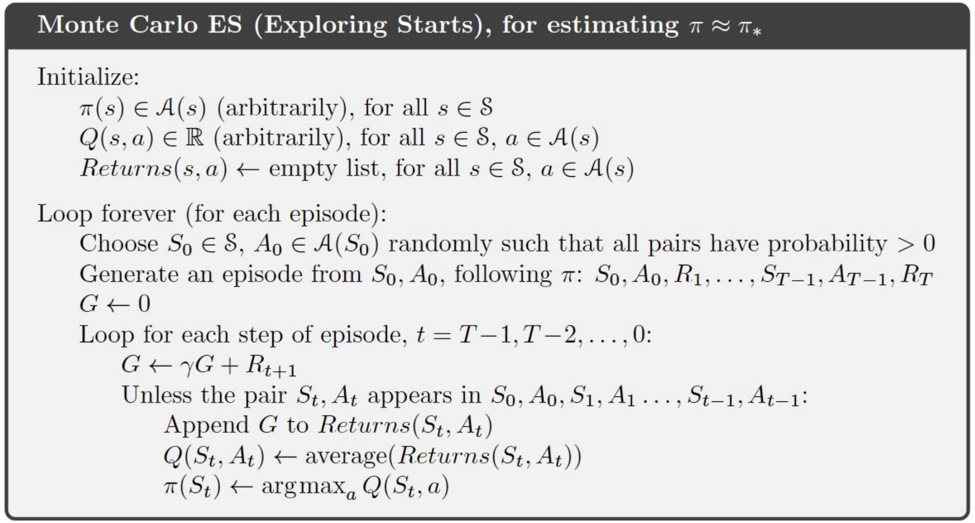
```
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
|   $\Delta \leftarrow 0$
|   Loop for each $s \in \mathcal{S}$:
|       $v \leftarrow V(s)$
|       $V(s) \leftarrow \max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
|       $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
    $\pi(s) = \operatorname{argmax}_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
```

Monte Carlo

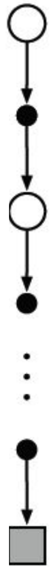
- We do not assume complete knowledge of the environment.
- Monte Carlo methods require only experience—sample sequences of states, actions, and rewards from actual or simulated interaction with an environment.
- Average returns observed after visits to a state
- Does not depend on estimates of other states (no bootstrapping)
- Get rid of exploring starts:
    - **on-policy** (e.g. epsilon greedy),
    - **off policy** (i.e. importance sampling)



```
First-visit MC prediction, for estimating $V \approx v_\pi$

Input: a policy $\pi$ to be evaluated
Initialize:
    $V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$
    $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):
    Generate an episode following $\pi$: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
        $G \leftarrow \gamma G + R_{t+1}$
        Unless $S_t$ appears in $S_0, S_1, \ldots, S_{t-1}$:
            Append $G$ to $Returns(S_t)$
            $V(S_t) \leftarrow \text{average}(Returns(S_t))$
```



```
Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize:
    $\pi(s) \in \mathcal{A}(s)$ (arbitrarily), for all $s \in \mathcal{S}$
    $Q(s, a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$
    $Returns(s, a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Loop forever (for each episode):
    Choose $S_0 \in \mathcal{S}$, $A_0 \in \mathcal{A}(S_0)$ randomly such that all pairs have probability $> 0$
    Generate an episode from $S_0, A_0$, following $\pi$: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
        $G \leftarrow \gamma G + R_{t+1}$
        Unless the pair $S_t, A_t$ appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:
            Append $G$ to $Returns(S_t, A_t)$
            $Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$
            $\pi(S_t) \leftarrow \text{argmax}_a Q(S_t, a)$
```



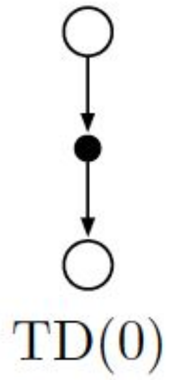
Temporal Difference

- We do not assume complete knowledge of the environment.
- TD methods require only experience—sample sequences of states, actions, and rewards from actual or simulated interaction with an environment.
- Bootstrap from estimates of other states
- Monte Carlo need to wait until end of episode, while TD(0) methods only need to wait one step
- **On-policy** (i.e. SARSA), **off-policy** (i.e. Q-learning)

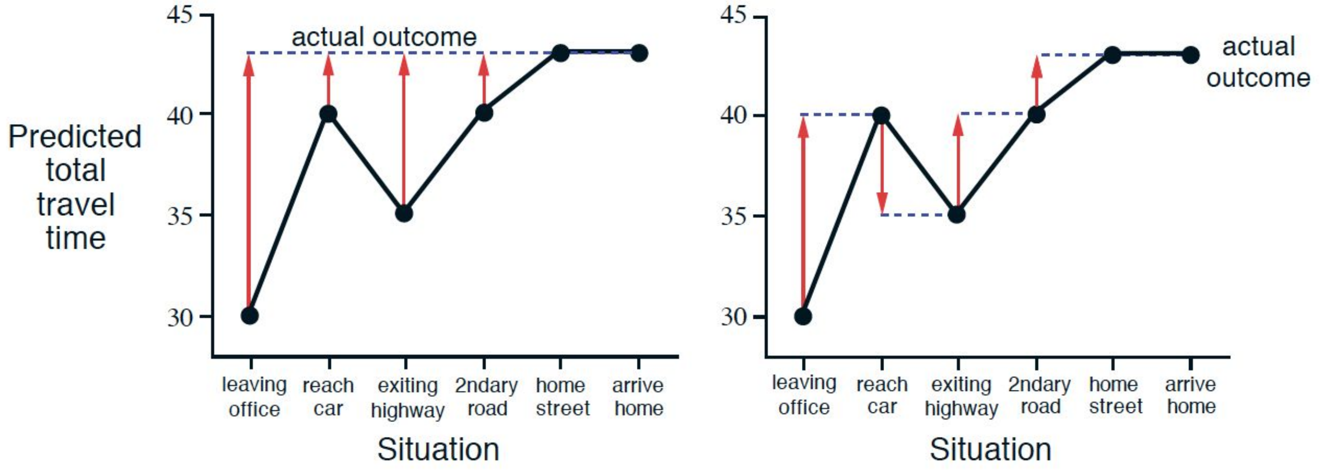
$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

Tabular TD(0) for estimating v_π

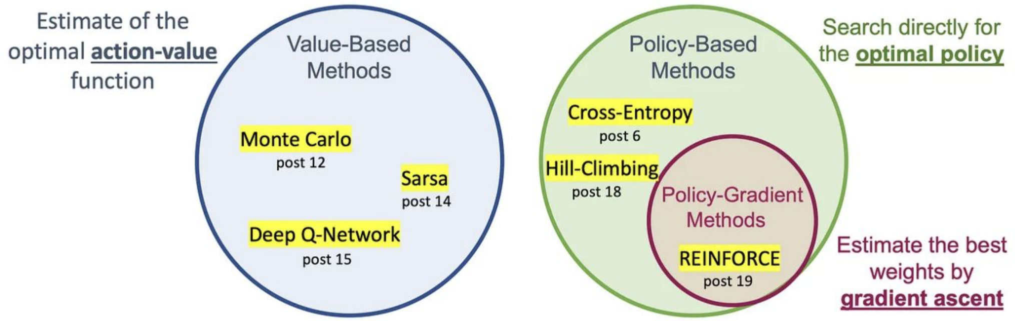
```
Input: the policy $\pi$ to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop for each episode:
    Initialize $S$
    Loop for each step of episode:
        $A \leftarrow$ action given by $\pi$ for $S$
        Take action $A$, observe $R$, $S'$
        $V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$
        $S \leftarrow S'$
    until $S$ is terminal
```



Monte Carlo vs Temporal Difference



Policy-Gradient Methods



- Value-based methods: learn a value function (an optimal value function leads to an optimal policy)
  - Goal: minimize the loss between the predicted and target value
  - Policy is implicit as it is generated directly from the value function (e.g. eps-greedy from Q-function)
  - Examples: Monte Carlo, DQN, SARSA
- Policy-based methods: learn to approximate optimal policy directly (without learning a value function)
  - Parameterize the policy, e.g. using a neural network
  - Policy outputs a probability distribution over actions (stochastic policy)
  - Goal: maximize the performance of the parameterized policy using gradient ascent

REINFORCE: Algorithm

REINFORCE, or Monte Carlo policy-gradient, uses an estimated return from an entire episode to update the policy parameter θ.

In a loop,

1. Use the policy π_θ to collect episode τ
2. Use the episode to estimate the gradient $g = \nabla \theta J(\theta)$

$$\nabla_\theta J(\theta) \approx \underbrace{\hat{g}}_{\text{Estimation of the gradient (given we use only one trajectory to estimate the gradient)}} = \sum_{t=0} \underbrace{\nabla_\theta \log \underbrace{\pi_\theta(a_t|s_t)}_{\text{Probability of the agent to select action at from state st given our policy}}}_{\text{Direction of the steepest increase of the (log) probability of selecting action at from state st}} \underbrace{R(\tau)}_{\text{Cumulative return}}$$

3. Update the weights of the policy: $\theta \leftarrow \theta + \alpha g$

REINFORCE: Problem

1. Let’s suppose we have a 3 armed bandit environment where the mean rewards for the arms are 10, 5, and 2.5 (with normally distributed noise with 0 mean and 1 variance).

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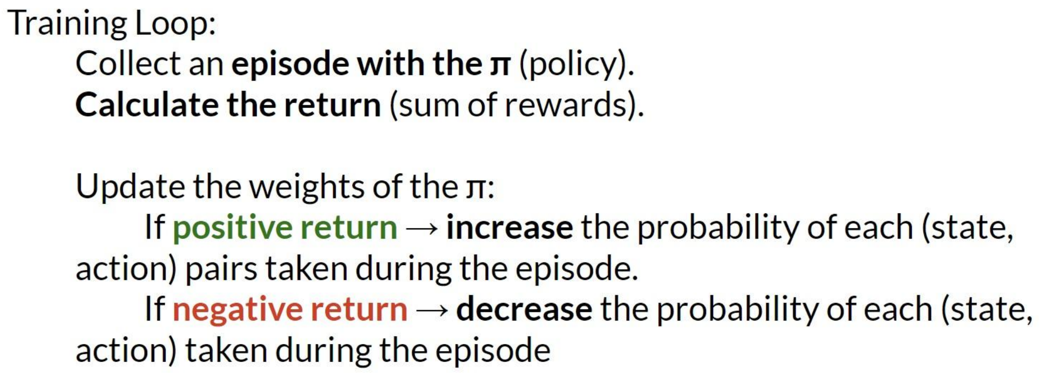

Training Loop:

Collect an **episode with the π** (policy).

Calculate the return (sum of rewards).

Update the weights of the π:

If positive return → **increase** the probability of each (state, action) pairs taken during the episode.

If negative return → **decrease** the probability of each (state, action) taken during the episode

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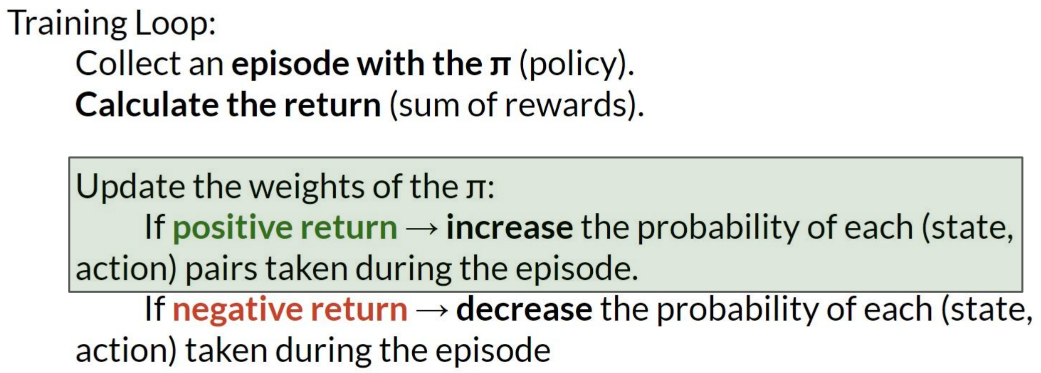

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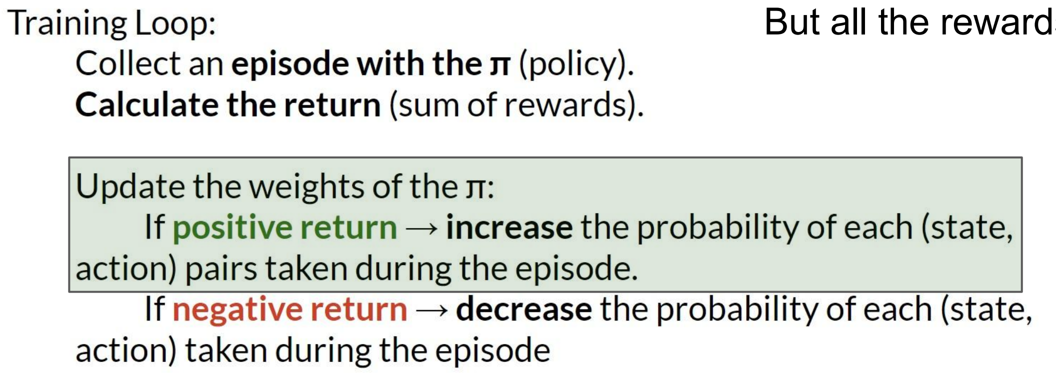

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But all the rewards are positive...

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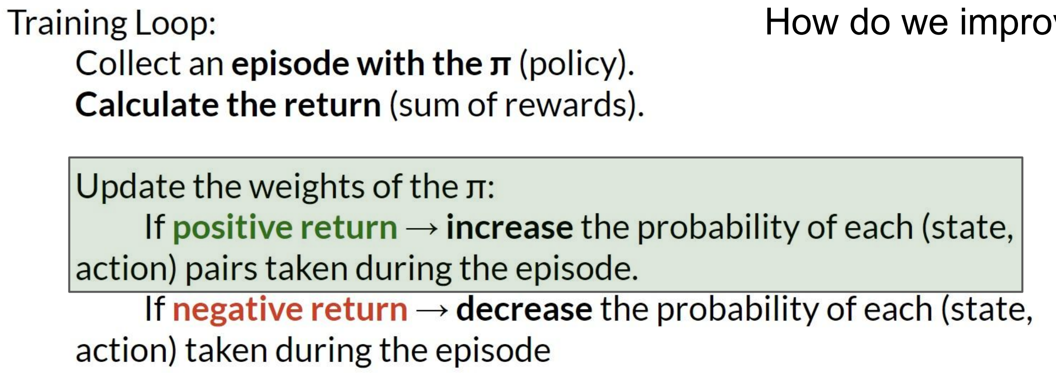

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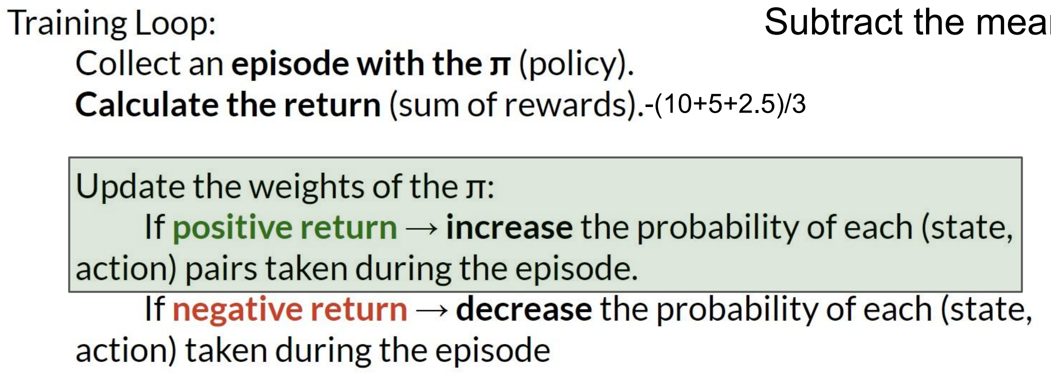
How do we improve this?

REINFORCE: Problem

1. Let’s suppose we have a 3 armed bandit environment where the mean rewards for the arms are 10, 5, and 2.5 (with normally distributed noise with 0 mean and 1 variance).
2. What would REINFORCE do?

Subtract the mean!



Training Loop:

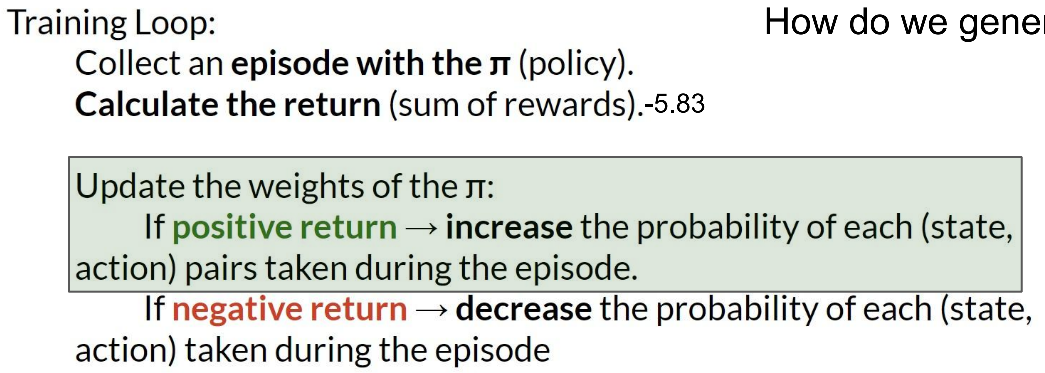
    Collect an **episode with the π** (policy).
    Calculate the return (sum of rewards).-(10+5+2.5)/3

    Update the weights of the π:
        If positive return → **increase** the probability of each (state, action) pairs taken during the episode.
        If negative return → **decrease** the probability of each (state, action) taken during the episode

REINFORCE: Problem

1. Let’s suppose we have a 3 armed bandit environment where the mean rewards for the arms are 10, 5, and 2.5 (with normally distributed noise with 0 mean and 1 variance).
2. What would REINFORCE do?

How do we generalize this?

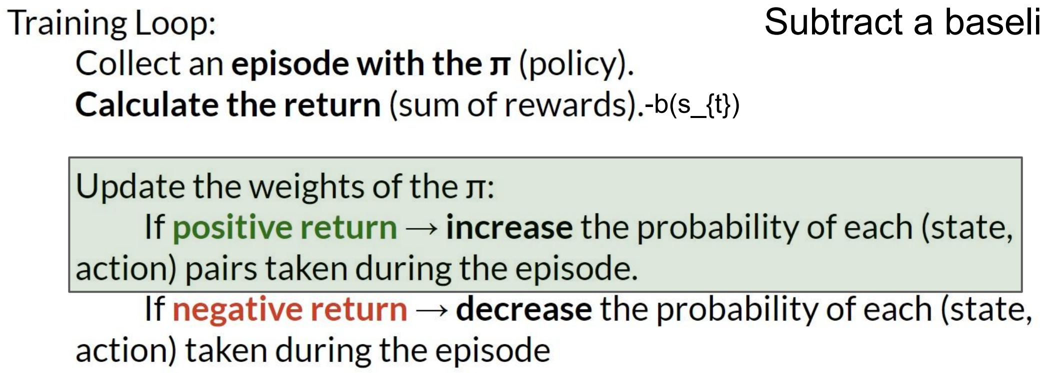


```
Training Loop:
    Collect an **episode with the π** (policy).
    **Calculate the return** (sum of rewards).-5.83

    Update the weights of the π:
        If positive return → **increase** the probability of each (state, action) pairs taken during the episode.
        If negative return → **decrease** the probability of each (state, action) taken during the episode
```

REINFORCE: Problem

1. Let’s suppose we have a 3 armed bandit environment where the mean rewards for the arms are 10, 5, and 2.5 (with normally distributed noise with 0 mean and 1 variance).
2. What would REINFORCE do?

Subtract a baseline!



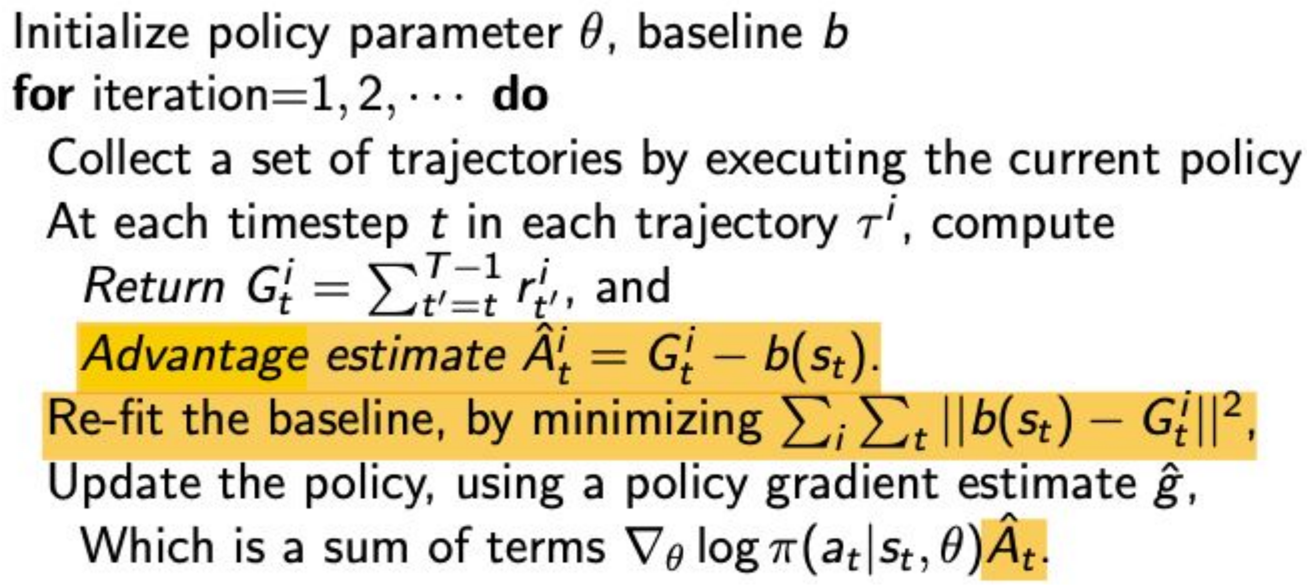
```
Training Loop:
    Collect an **episode with the π** (policy).
    **Calculate the return** (sum of rewards).-b(s_{t})

    Update the weights of the π:
        If positive return → **increase** the probability of each (state, action) pairs taken during the episode.
        If negative return → **decrease** the probability of each (state, action) taken during the episode
```

REINFORCE - Baseline: Algorithm

1. What did we make? (Hint: Read the slide title)

REINFORCE - Baseline: Algorithm

1. What did we make?



```
Initialize policy parameter $\theta$, baseline $b$
for iteration$=1, 2, \cdots$ do
  Collect a set of trajectories by executing the current policy
  At each timestep $t$ in each trajectory $\tau^i$, compute
    Return $G_t^i = \sum_{t'=t}^{T-1} r_{t'}^i$, and
    Advantage estimate $\hat{A}_t^i = G_t^i - b(s_t)$.
  Re-fit the baseline, by minimizing $\sum_i \sum_t ||b(s_t) - G_t^i||^2$,
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
```

REINFORCE - Baseline: Problem

1. Let’s suppose we have an environment where we only get rewards at the end of the game (i.e. tic tac toe, chess, shogi, go, etc.)

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REINFORCE - Baseline: Problem

1. Let’s suppose we have an environment where we only get rewards at the end of the game (i.e. tic tac toe, chess, shogi, go, etc.)
2. Did my move at the beginning of the game **actually** have a **significant influence** on the outcome of the game?

$$G_t = \overset{0}{R_t} + \gamma \overset{0}{R_{t+1}} + \gamma^2 \overset{0}{R_{t+2}} + \ldots \quad \overset{1}{}$$

REINFORCE - Baseline: Problem

1. Let’s suppose we have an environment where we only get rewards at the end of the game (i.e. tic tac toe, chess, shogi, go, etc.)
2. Did my move at the beginning of the game **actually** have a **significant influence** on the outcome of the game?

$$G_t = \overset{0}{R_t} + \gamma \overset{0}{R_{t+1}} + \gamma^2 \overset{0}{R_{t+2}} + \ldots \quad 1$$

3. How did we solve this problem with Value Based methods?

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3. How did we solve this problem with Value Based methods?

Bootstrapping!

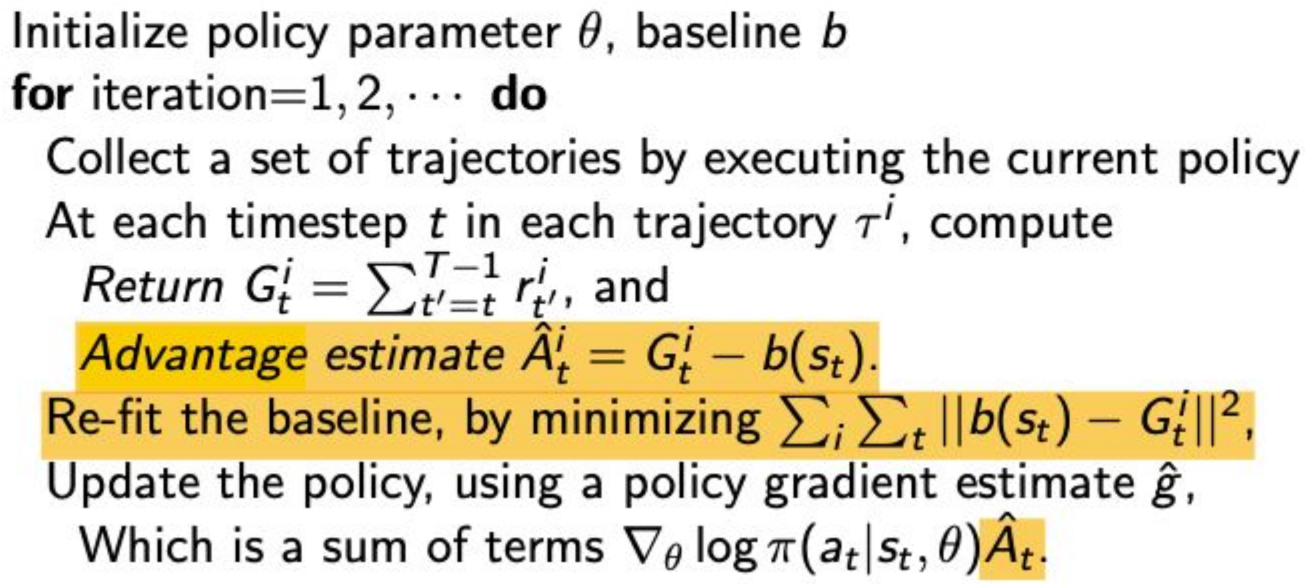
REINFORCE - Baseline: Problem

1. Let’s change the advantage function

REINFORCE - Baseline: Problem

1. Let’s change the advantage function

We had this:

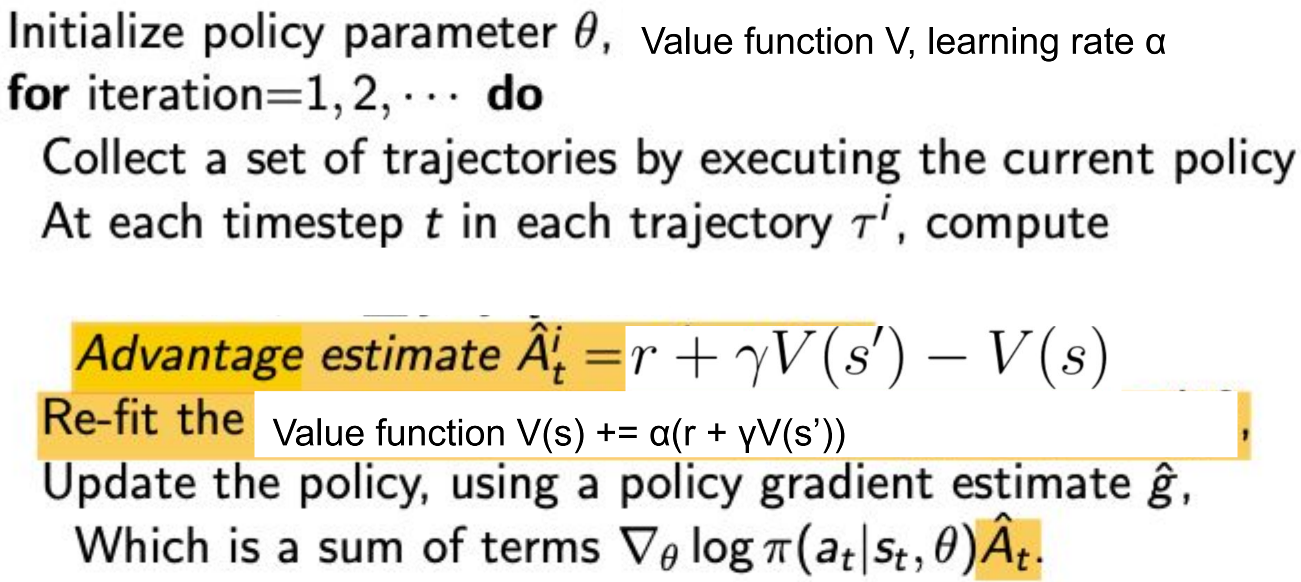


```
Initialize policy parameter $\theta$, baseline $b$
for iteration=$1, 2, \cdots$ do
  Collect a set of trajectories by executing the current policy
  At each timestep $t$ in each trajectory $\tau^i$, compute
    Return $G_t^i = \sum_{t'=t}^{T-1} r_{t'}^i$, and
    Advantage estimate $\hat{A}_t^i = G_t^i - b(s_t)$.
  Re-fit the baseline, by minimizing $\sum_i \sum_t ||b(s_t) - G_t^i||^2$,
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
```

REINFORCE - Baseline: Problem

1. Let’s change the advantage function

Now we have this:

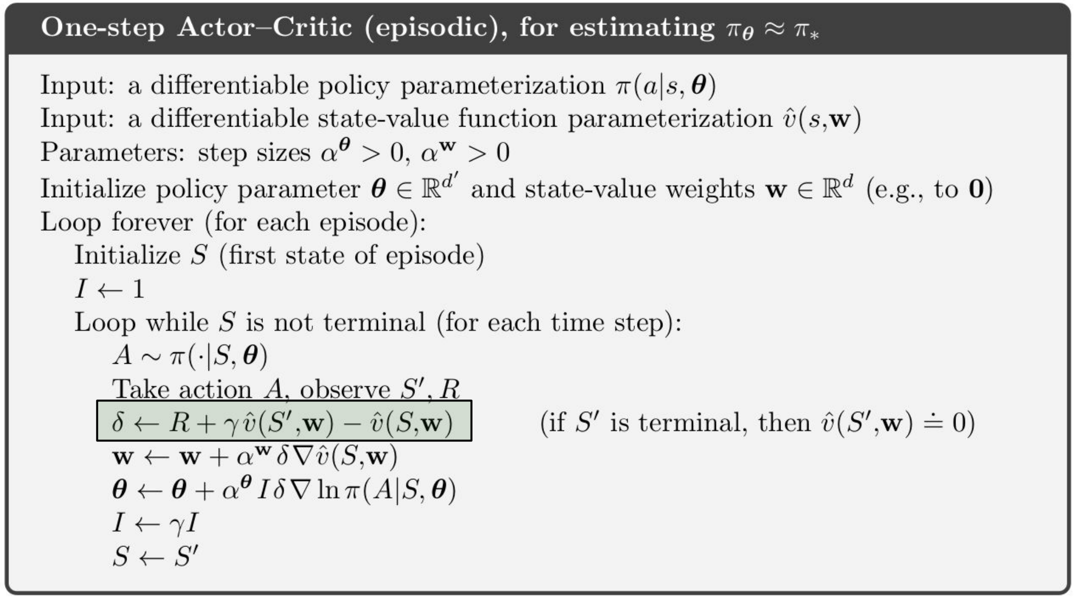


```
Initialize policy parameter $\theta$, Value function V, learning rate α
for iteration=$1, 2, \cdots$ do
  Collect a set of trajectories by executing the current policy
  At each timestep $t$ in each trajectory $\tau^i$, compute
    Advantage estimate $\hat{A}_t^i = r + \gamma V(s') - V(s)$
  Re-fit the Value function V(s) += α(r + γV(s’)),
  Update the policy, using a policy gradient estimate $\hat{g}$,
    Which is a sum of terms $\nabla_\theta \log \pi(a_t|s_t, \theta)\hat{A}_t$.
```

REINFORCE - Baseline: Problem

1. Let’s change the advantage function

A bit neater now:



One-step Actor–Critic (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

```
Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
    Initialize $S$ (first state of episode)
    $I \leftarrow 1$
    Loop while $S$ is not terminal (for each time step):
        $A \sim \pi(\cdot|S, \boldsymbol{\theta})$
        Take action $A$, observe $S', R$
        $\boxed{\delta \leftarrow R + \gamma \hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})}$        (if $S'$ is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S,\mathbf{w})$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I \delta \nabla \ln \pi(A|S, \boldsymbol{\theta})$
        $I \leftarrow \gamma I$
        $S \leftarrow S'$
```

Advantage Actor Critic (A2C):

1. What did we make this time?

Advantage Actor Critic (A2C): Differences

1. What did we make this time?
2. Let's distill the changes we made

REINFORCE

$$\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s})\, G_t$$

$\longrightarrow$

REINFORCE - Baseline

$$\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) A(\mathbf{s}, \mathbf{a})$$

A(s,a) = G_{t} - b(s_{t})

REINFORCE - Baseline

A(s,a) = G_{t} - b(s_{t})

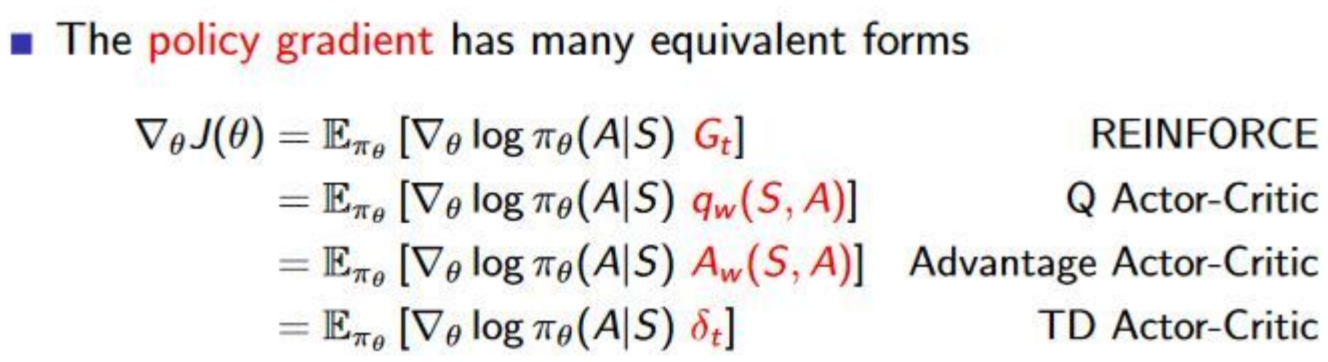
$\longrightarrow$

A2C

$$A(s, a) = \underbrace{r + \gamma V(s') - V(s)}_{\text{TD Error}}$$

Actor Critic vs Advantage Actor Critic

1. Actor Critic
   a. We don’t directly use the discounted cumulative rewards to calculate the policy update
2. Advantage Actor Critic
   a. We use the advantage function (or an approximation) to calculate the policy update



- The policy gradient has many equivalent forms

$$\begin{aligned}
\nabla_\theta J(\theta) &= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(A|S)\; G_t\right] && \text{REINFORCE} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(A|S)\; q_w(S,A)\right] && \text{Q Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(A|S)\; A_w(S,A)\right] && \text{Advantage Actor-Critic} \\
&= \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta(A|S)\; \delta_t\right] && \text{TD Actor-Critic}
\end{aligned}$$

Policy-based methods, pros and cons

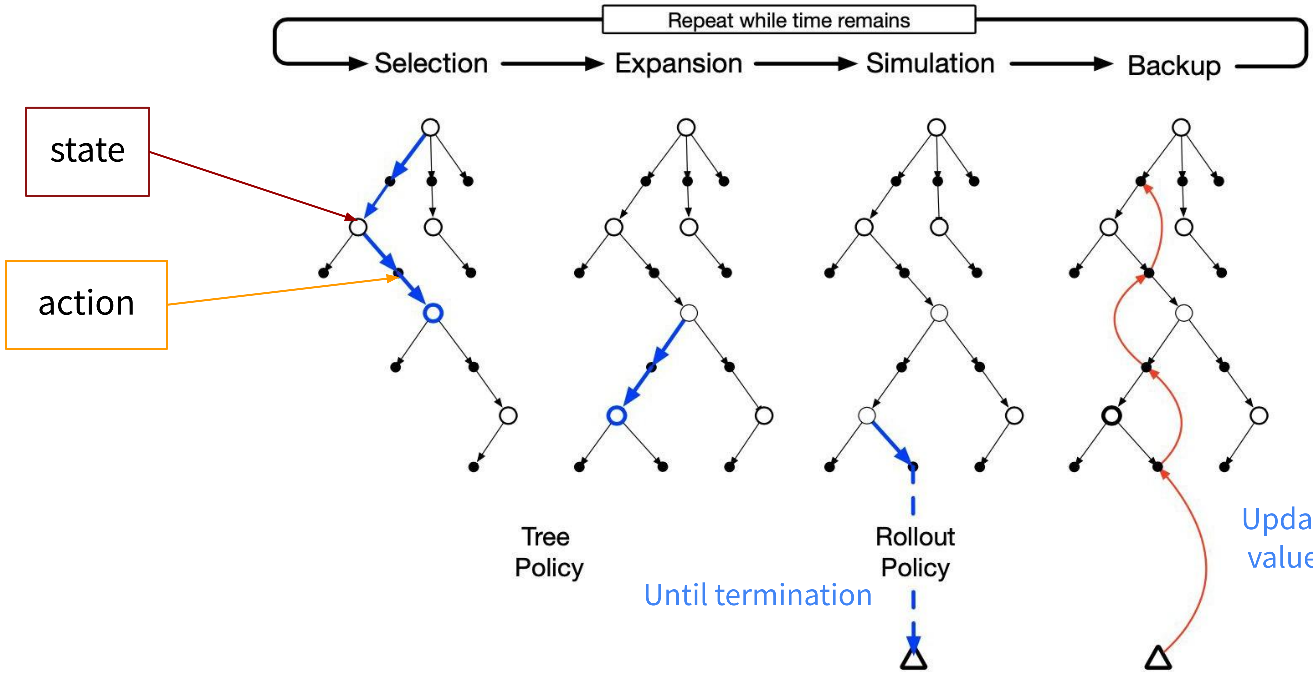
Pros

- We can estimate the policy directly without storing additional data
- Policy-gradient methods can learn a stochastic policy
    - We don’t need to implement an exploration/exploitation trade-off by hand
- More effective in high-dimensional action spaces and continuous action spaces 🤔
- Better convergence properties 🤔

Cons

- Converges to a local maximum sometimes
- Slower, step-by-step: it can take longer to train (inefficient)
- Gradient estimate is very noisy: there is a possibility that the collected trajectory may not be representative of the policy

Monte Carlo Tree Search



Monte Carlo Tree Search: Benefits

When to use MCTS over learning algorithms?

- More useful if you have limited amount of time
- Access to internal model
- Size or dynamic nature of the state-action space (in MCTS, the state action space size doesn’t matter because it only explores the best actions)

Deep Q-Network

Algorithm 4 DQN

```
procedure DQN
    Initialize network $Q_\omega$ and $Q_{\text{target}}$ as a clone of $Q_\omega$
    Initialize replay buffer $R$ and burn in with trajectories followed by random policy
    Initialize $c = 0$
    repeat for $E$ training episodes:
        Initialize $S_0$
        for $t = 0, 1, \ldots, T-1$:
            $a_t = \begin{cases} \arg\max_a Q_\omega(s_t, a) & \text{with probability } 1-\epsilon \\ \text{Random action} & \text{otherwise} \end{cases}$
            Take $a_t$ and observe $r_t, s_{t+1}$
            Store $(s_t, a_t, r_t, s_{t+1})$ in $R$
            Sample minibatch of $(s_i, a_i, r_i, s_{i+1})$ with size $N$ from $R$
            $y_i = \begin{cases} r_i & s_{i+1} \text{ is terminal} \\ r_i + \gamma \max_a Q_{\text{target}}(s_{i+1}, a) & \text{otherwise} \end{cases}$
            $L(\omega) = \frac{1}{N} \sum_{i=0}^{N-1} (y_i - Q_\omega(s_i, a_i))^2$
            Update $Q_\omega$ using Adam $(\nabla_\omega L(\omega))$
            $c = c + 1$
            Replace $Q_{\text{target}}$ with current $Q_\omega$ if $c \% 50 = 0$
end procedure
```

Sampling: we perform actions and store the observed experience tuples in a replay memory

Training: select a small batch of tuples randomly and learn from this batch using a gradient descent update step

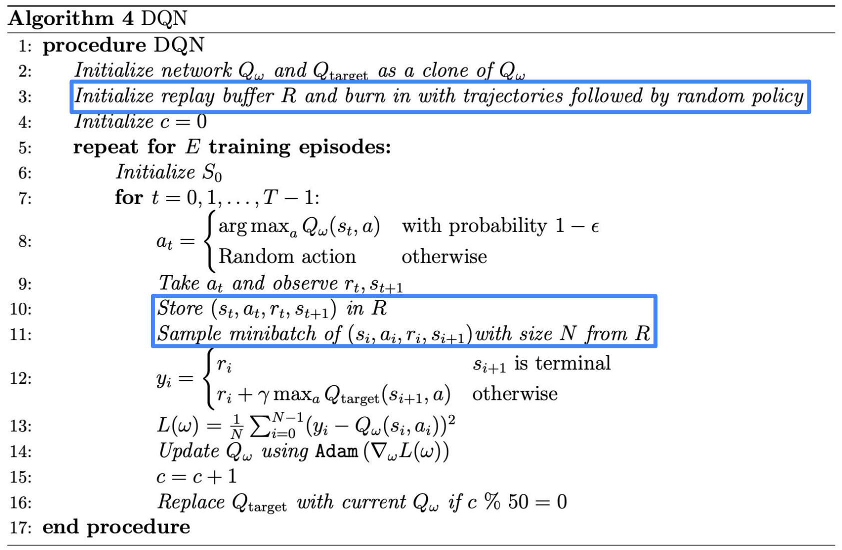
Deep Q-Network

Because deep Q-learning combines a non-linear Q-value function (Neural network) with bootstrapping (when we update targets with existing estimates and not an actual complete return), it might suffer from instability.

To help us stabilize the training, we implement three different solutions:

1. **Experience Replay** to make more efficient use of experiences.
2. **Fixed Q-Target** to stabilize the training.
3. Double Deep Q-Learning, to handle the problem of the overestimation of Q-values.

Deep Q-Learning: Experience Replay



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        for $t = 0, 1, \ldots, T-1$:
            $a_t = \begin{cases} \arg\max_a Q_\omega(s_t, a) & \text{with probability } 1-\epsilon \\ \text{Random action} & \text{otherwise} \end{cases}$
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            $y_i = \begin{cases} r_i & s_{i+1} \text{ is terminal} \\ r_i + \gamma \max_a Q_{\text{target}}(s_{i+1}, a) & \text{otherwise} \end{cases}$
            $L(\omega) = \frac{1}{N} \sum_{i=0}^{N-1} (y_i - Q_\omega(s_i, a_i))^2$
            Update $Q_\omega$ using Adam $(\nabla_\omega L(\omega))$
            $c = c + 1$
            Replace $Q_{\text{target}}$ with current $Q_\omega$ if $c \% 50 = 0$
end procedure
```

Uses the experiences of the training more efficiently (we can use a replay buffer that saves experience samples that we can reuse during sampling)

- Agent can learn from the same experience multiple times!

Avoid forgetting previous experiences and reduce the correlation between experiences

- if we give sequential samples of experiences to our neural network is that it tends to forget the previous experiences as it gets new experiences

By randomly sampling experiences, we remove correlation in the observation sequences to avoid actin values from oscillating or diverging catastrophically.

Deep Q-Learning: Fixed Q-Target

$$L = \Big(\underbrace{\mathrm{sg}(R_{t+1} + \gamma \max_{A_{t+1}} q(S_{t+1}, A_{t+1}, w))}_{\text{Target value}} - \underbrace{q(S_t, A_t, w)}_{\text{Prediction}}\Big)^2$$

- **Problem**: at every step of training, both our Q-values and target values shift (nonstationary targets)
  - Where the most instability comes from
  - Updating the network weights changes the target value, which requires more updates
  - Unintended generalization to other states S' can lead to error propagation
- **Solution**: use a separate network with fixed parameters to estimate the TD target and compy the parameters from our Deep Q-Network every *c* steps
  - For *c* steps, the target network is fixed, after that you update the target network once and continue to update your value function for another *c* steps, repeat the process
  - Network has more time to fit targets accurately before they change
  - Slows down training, but not too many alternatives (recently: functional regularization)

Deep Q-Learning: Fixed Q-Target

Algorithm 4 DQN

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            $c = c + 1$
            Replace $Q_{\text{target}}$ with current $Q_\omega$ if $c \,\%\, 50 = 0$
end procedure
```

Big Picture Table

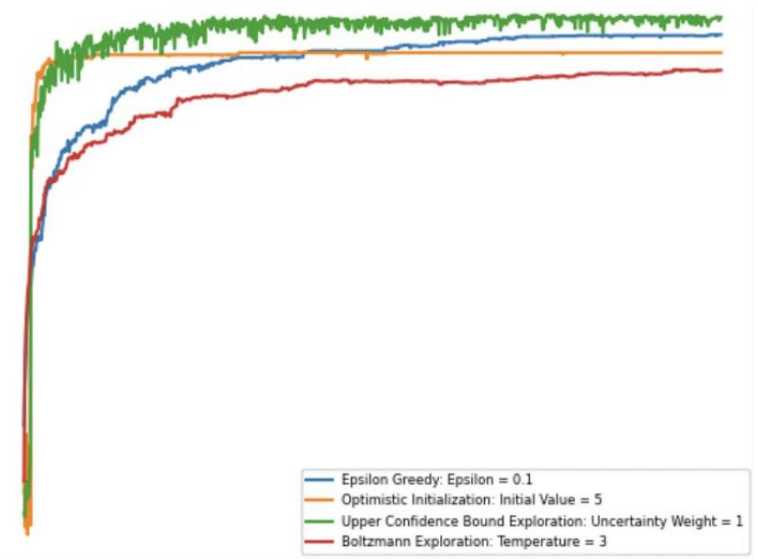
Method	On/Off Policy?	Bootstraps?
Monte Carlo Methods	On*	N
SARSA	On	Y
Expected SARSA	Off	Y
Q-Learning	Off	Y
REINFORCE	On	N
Actor Critic, A2C	On*	Y

* can be made off policy with importance sampling

Practice Questions

Question 1) Bandits

Why does the expected reward curve for UCB look so noisy, especially compared to epsilon-greedy or Boltzmann?



Solution 1) Bandits

At every timestep, the UCB policy is deterministic and rapidly switches between actions in order to explore. In contrast, Boltzmann uses a stochastic policy, so its expected reward changes smoothly as the action distribution changes.

Question 2) Scaling Rewards

Let’s say that we have 2 3-armed bandits:

- Mean rewards (-1, 0, 1) and noisy Gaussian reward with variance 1
- Mean rewards (-10, 0, 10) and noisy Gaussian reward with variance 100

For the same random seed, does epsilon-greedy take the same sequence of actions? How about Boltzmann exploration?

Solution 2) Scaling Rewards

Epsilon-greedy is scale-invariant; only the maximum Q-value determines the policy. Therefore, it takes the same sequence of actions on both. However, Boltzmann exploration depends on the gap between the Q-values, so it is much more stochastic in the first bandit problem than in the second.

$$\pi = p(A_t = a) = \frac{exp(\tau \hat{q}_{a,t})}{\sum_{a' \in \mathcal{A}} exp(\tau \hat{q}_{a,t})}$$

Question 3) Markov Decision Processes

For a finite MDP, let's say we have initial state distribution $\rho_0(s)$, transition function $T(s'|s,a)$, and fixed policy $\pi(a|s)$. What's the probability of seeing some sequence of states $s_0, s_1, \ldots, s_n$?

Solution 3) Markov Decision Processes

$$
\begin{aligned}
p(s_0, s_1, \ldots, s_n) &= p(s_0) \prod_{t=1}^{n} p(s_t | s_{t-1}) && \text{(future indep. of past, given current state)} \\
&= p(s_0) \prod_{t=1}^{n} \left(\sum_{a_{t-1}} p(a_{t-1} | s_{t-1}) p(s_t | s_{t-1}, a_{t-1}) \right) && \text{(unmarginalize action)} \\
&= \rho_0(s_0) \prod_{t=1}^{n} \left(\sum_{a_{t-1}} \pi(a_{t-1} | s_{t-1}) T(s_t | s_{t-1}, a_{t-1}) \right) && \text{(applying policy and dynamics)}
\end{aligned}
$$

Question 4) Comparing SARSA and Q-Learning

Given some trajectory: $(S_0, A_0, R_0), (S_1, A_1, R_1), .. (S_N, A_N, R_N)$

We can define an update target for Q-values at step 0:

$$Q^{\pi,\gamma}_{target}(S_0, A_0) = R_0 + \gamma R_1 + \gamma^2 R_2 + \cdots \gamma^N R_N$$

Using V(s), apply bootstrapping for 2-step returns.

$$Q^{\pi,\gamma}_{target}(S_0, A_0) = R_0 + \gamma R_1 + \gamma^2 V^{\pi,\gamma}(S_2)$$

Do the same, but in terms of Q(s, a).

$$Q^{\pi,\gamma}_{target}(S_0, A_0) = R_0 + \gamma R_1 + \gamma^2 Q^{\pi,\gamma}(S_2, A_2)$$

Do the same, assuming the policy takes optimal actions with respect to its Q values.

$$Q^{\pi,\gamma}_{target}(S_0, A_0) = R_0 + \gamma R_1 + \gamma^2 max_{a' \in A} Q^{\pi,\gamma}(S_2, a')$$

Question 5) Comparing SARSA and Q-Learning

At each timestep of the policy iteration algorithm, the expected reward of the current policy is guaranteed to improve or remain the same.

Answer 5) Value and Policy Iteration

True, this is the Policy Improvement Theorem.

Question 6) SARSA

Was SARSA designed to learn Q-values using samples from a replay buffer of transitions collected from old policies? What about expected SARSA?

Answer 6) SARSA

SARSA: $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$

Expected SARSA: $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t)\Big]$

SARSA is on-policy: A' is supposed to be drawn from the policy. (It can be extended to use a replay buffer, but this isn't in the original formulation). In contrast, expected SARSA is designed to use (s, a, s', r) from any source to perform updates.

Question 7) MCTS & DQN

Which of the following statements about MCTS and DQNs is incorrect:

A) MCTS uses a tabular representations of action-values whereas DQNs uses a functional approximation.

B) Agents using DQNs are faster at choosing actions compared to those using MCTS.

C) DQNs and MCTS are both on-policy.

D) MCTS have been shown to outperform comparable DQNs on some tasks.

Answer 7) MCTS & DQN

C) DQNs and MCTS are both on-policy.

Questions?