Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

# Multi-armed bandits

Fall 2021, CMU 10-703

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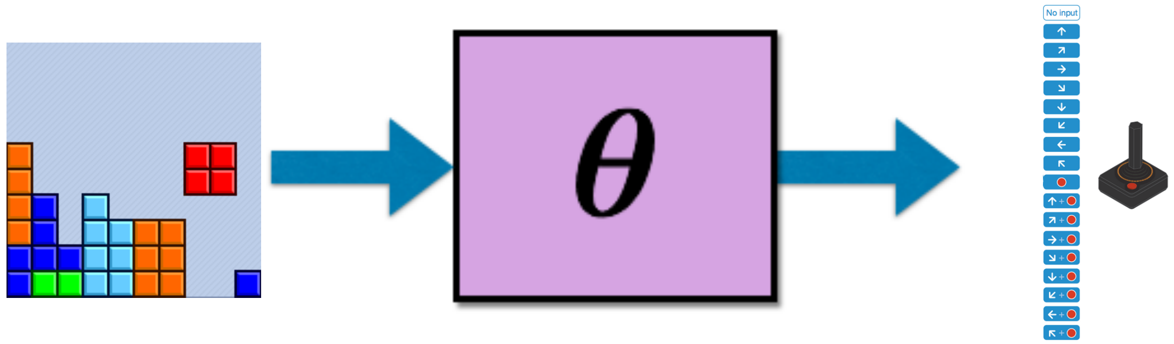
# Used Materials

- Disclaimer: Some material and slides for this lecture were borrowed from Rich Sutton’s lecture on multi-armed bandits.

# LL: Reinforcement Learning




$\pi(a \mid s, \theta)$

Given an initial state distribution $\mu_0(s_0)$, estimate parameters $\theta$ of a policy $\pi_\theta$ so that, the trajectories $\tau$ sampled from this policy have maximum returns, i.e., sum of rewards $R(\tau)$.

$$\max_{\theta} . \quad U(\theta) = \mathbb{E}_{\tau \sim \pi_\theta} \left[ R(\tau) \mid \pi_\theta, \mu_0(s_0) \right]$$

$\tau$ : trajectory, a sequence of state, action, rewards, a game fragment or a full game:

$$\tau : s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots s_T, a_T, r_T$$

$R(\tau)$ : reward of a trajectory: (discounted) sum of the rewards of the individual state/actions

$$R(\tau) = \sum_{t=1}^{T} r_t$$

# This lecture - Motivation

Learning to act in a non-sequential (single action) setups:

- Each action results in an immediate reward.
- We want to choose actions that maximize our immediate reward in expectation.
    - Q: Why in expectation?
    - A: Because rewards are not deterministic.
- For example, displaying an advertisement can generate different click rates in different days. Actions: the advertisements to be displayed, Rewards: the user click rate. We want to pick the advertisement that maximizes the click rate on average

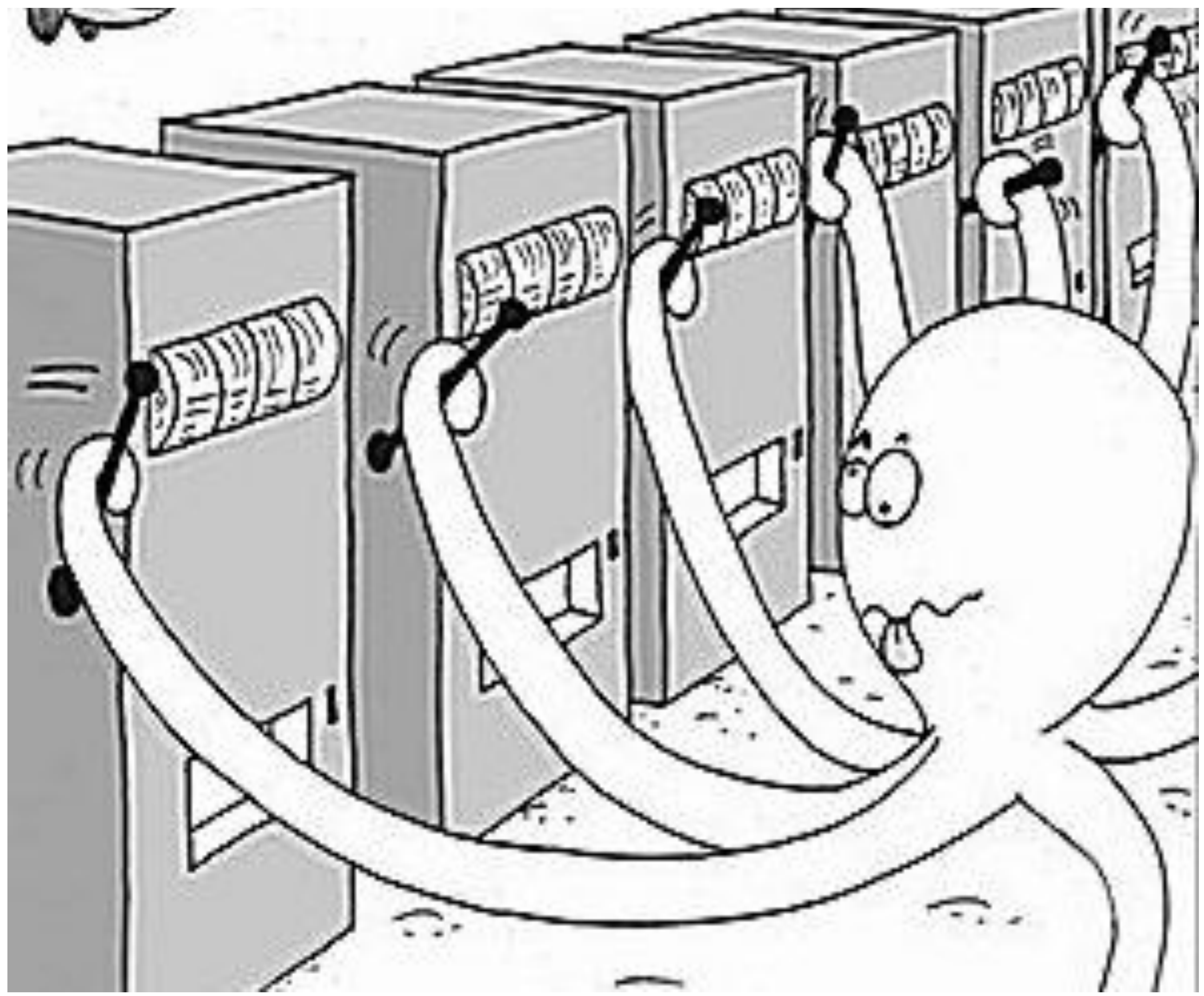
# Multi-Armed Bandits

One-armed bandit= Slot machine (English slang)


source: infoslotmachine.com

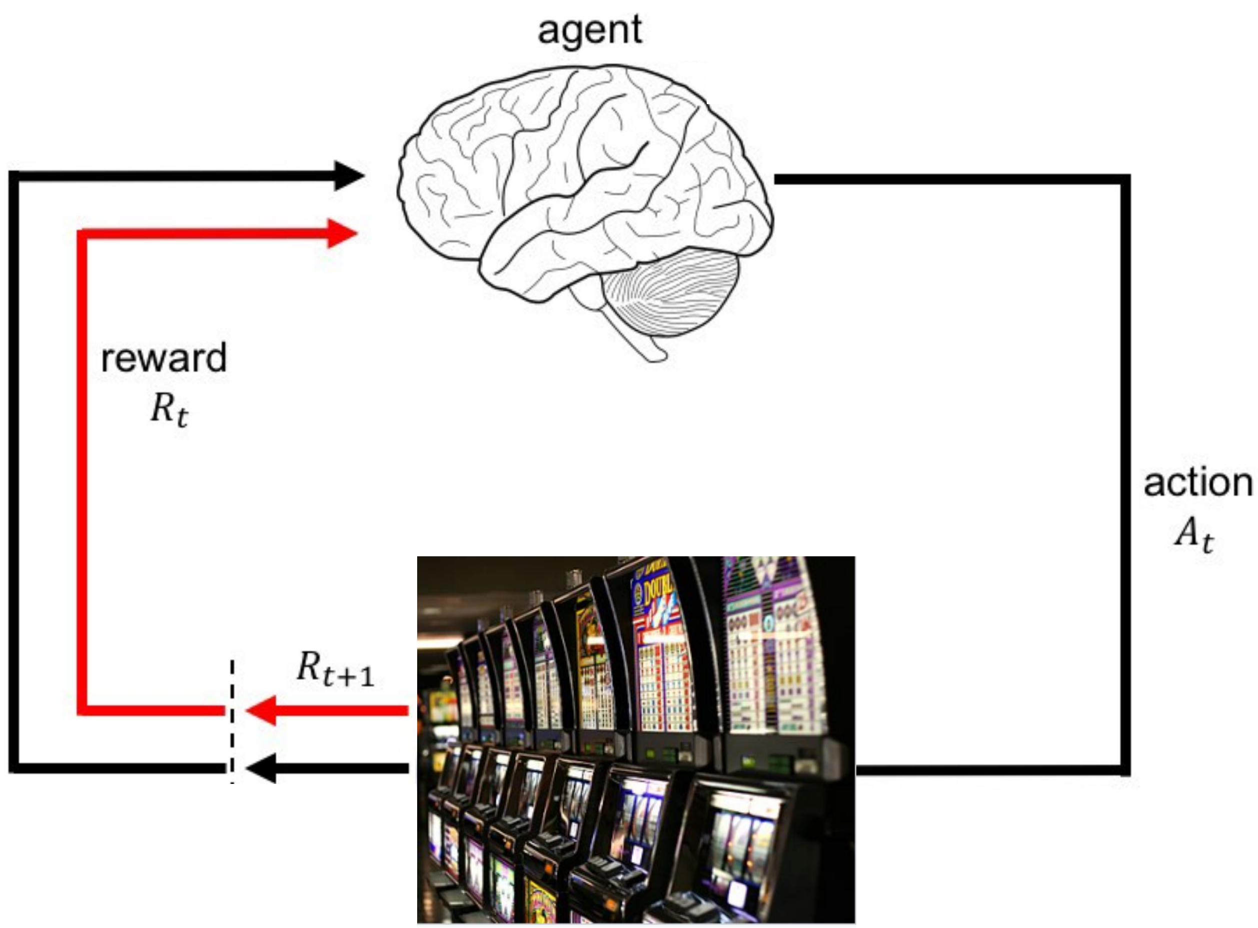
# Multi-Armed Bandits

Multi-Armed bandit = Multiple Slot Machine



source: Microsoft Research

# Multi-Armed Bandits




$$A_t, R_{t+1}, A_{t+1}, R_{t+2}, A_{t+2}, A_{t+3}, R_{t+3}, \ldots$$

The state does not change! (a.k.a. stateless)

# Multi-Armed Bandit Problem

At each timestep $t$ the agent chooses one of the K arms and plays it.

The $k$ th arm produces reward $r_{k,t}$ when played at timestep $t$ .

The rewards $r_{k,t}$ are drawn from a probability distribution $\mathcal{P}_k$ with mean $\mu_k$ .

The agent does not know neither the full arm reward distributions neither their means.


source: Pandey et al.'s slide

Agent's Objective: Maximize cumulative rewards (over a finite or infinite horizon).

I can maximize cumulative rewards over a finite or infinite horizon if i just play the arm with the highest mean reward $\mu_k$ each time. (but i do not know those..)

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source: Pandey et al.'s slide

Definition: The action-value for action $\alpha$ (here arm k) is its mean reward:

$$q_*(a) \doteq \mathbb{E}[R_t | A_t = a]$$

# The Exploration/Exploitation Dilemma

- Suppose you form estimates

$$Q_t(a) \approx q_*(a), \quad \forall a$$ *action-value estimates*

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- You can't do both, but you need to do both

- You can never stop exploring, but maybe you should explore less with time.

# Exploration vs Exploitation Dilemma

- Online decision-making involves a fundamental choice:
  - Exploitation: Make the best decision given current information
  - Exploration: Gather more information
- The best long-term strategy may involve short-term sacrifices
- Gather enough information to make the best overall decisions

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- The best long-term strategy may involve short-term sacrifices
- Gather enough information to make the best overall decisions
- The exploration/exploitation dilemma is not a problem encountered in computational RL or deep RL: It is a fundamental problem in decision making of any intelligent agent.

# Exploration vs. Exploitation Dilemma

- Restaurant Selection
  - Exploitation: Go to your favorite restaurant
  - Exploration: Try a new restaurant
- Oil Drilling
  - Exploitation: Drill at the best known location
  - Exploration: Drill at a new location
- Game Playing
  - Exploitation: Play the move you believe is best
  - Exploration: Play an experimental move

# Example: Bernoulli Bandits

Recall: The Bernoulli distribution is the discrete probability distribution of a random variable which takes the value 1 with probability p and the value 0 with probability q=1-p, that is, the probability distribution of any single experiment that asks a yes–no question.

- Each action (arm when played) results in success or failure, rewards are binary.
- Mean reward for each arm represents the probability of success.
- Action (arm) $k \in \{1 ... K\}$ produces a success with probability $\theta_k \in [0,1]$.


$\theta_1$

win 0.6
of time


$\theta_2$

win 0.4
of time

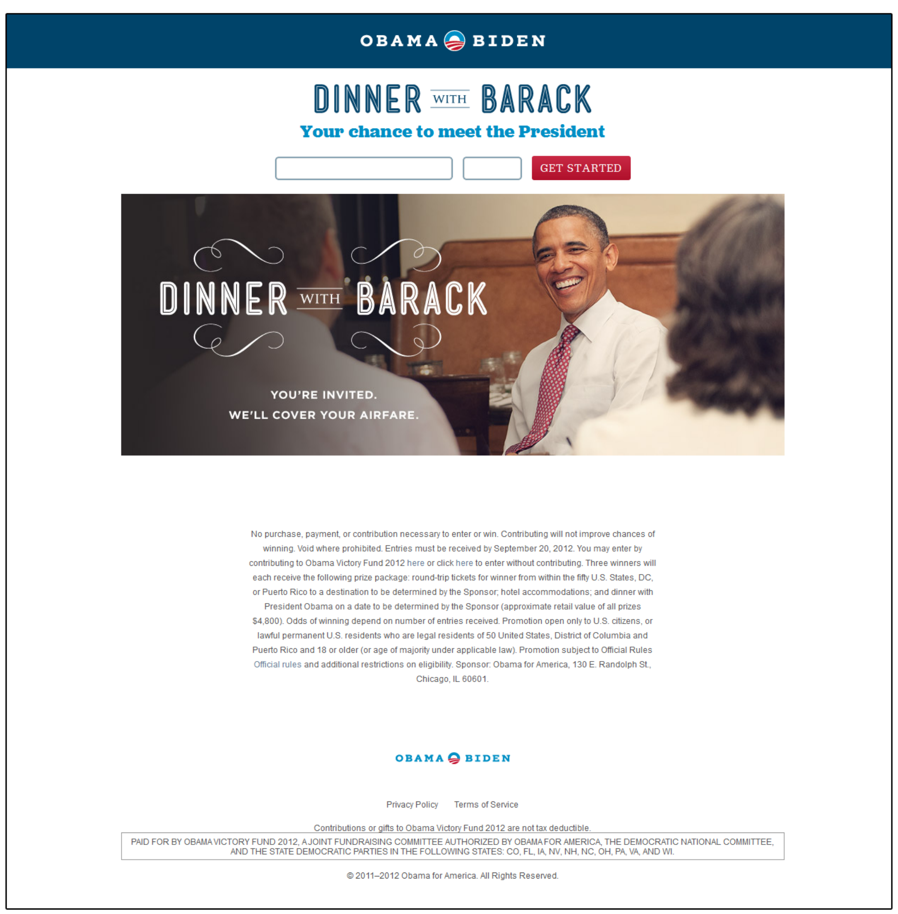

$\theta_3$

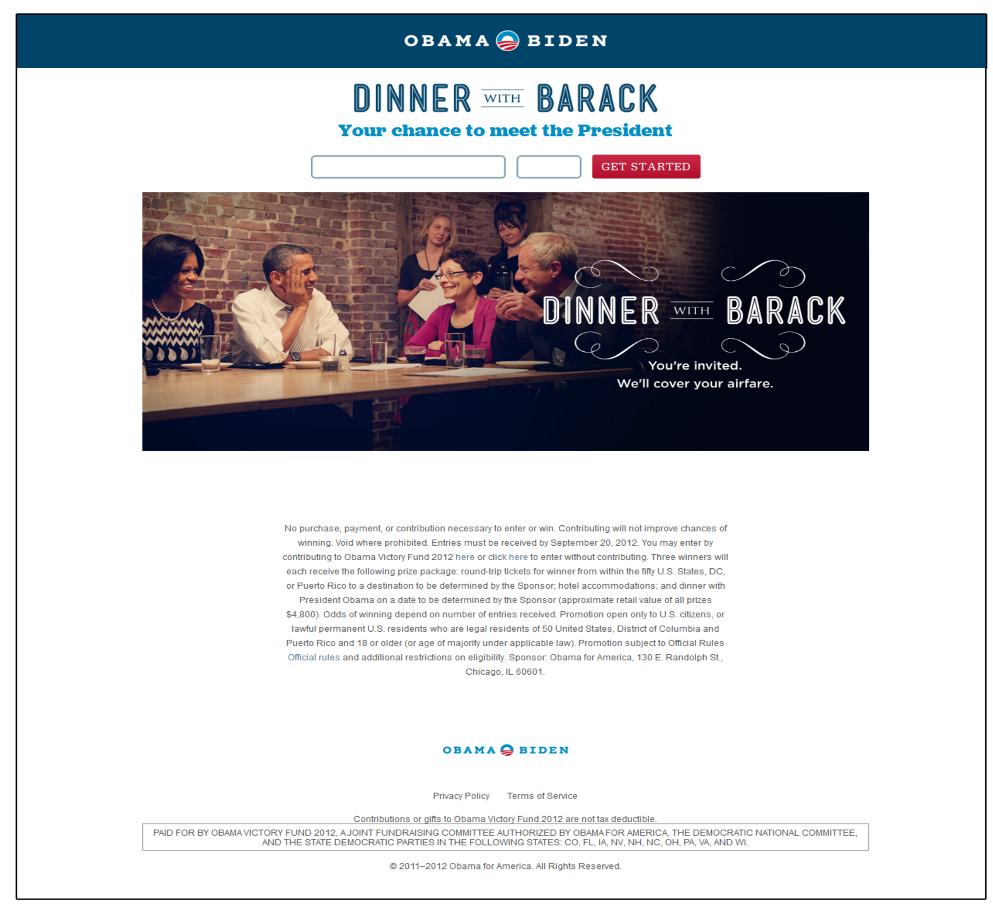
win 0.45
of time

# Real world motivation: content presentation

We have two variations of content of a webpage, A and B, and we want to decide which one to display to engage more users.

- Two arm bandits: each arm corresponds to a content variation shown to users (not necessarily the same user).
- Reward: 1 if the user clicks, 0 otherwise.
- Mean reward (success probability) for each invitation: the click-through-rate, the percentage of users that would click on it

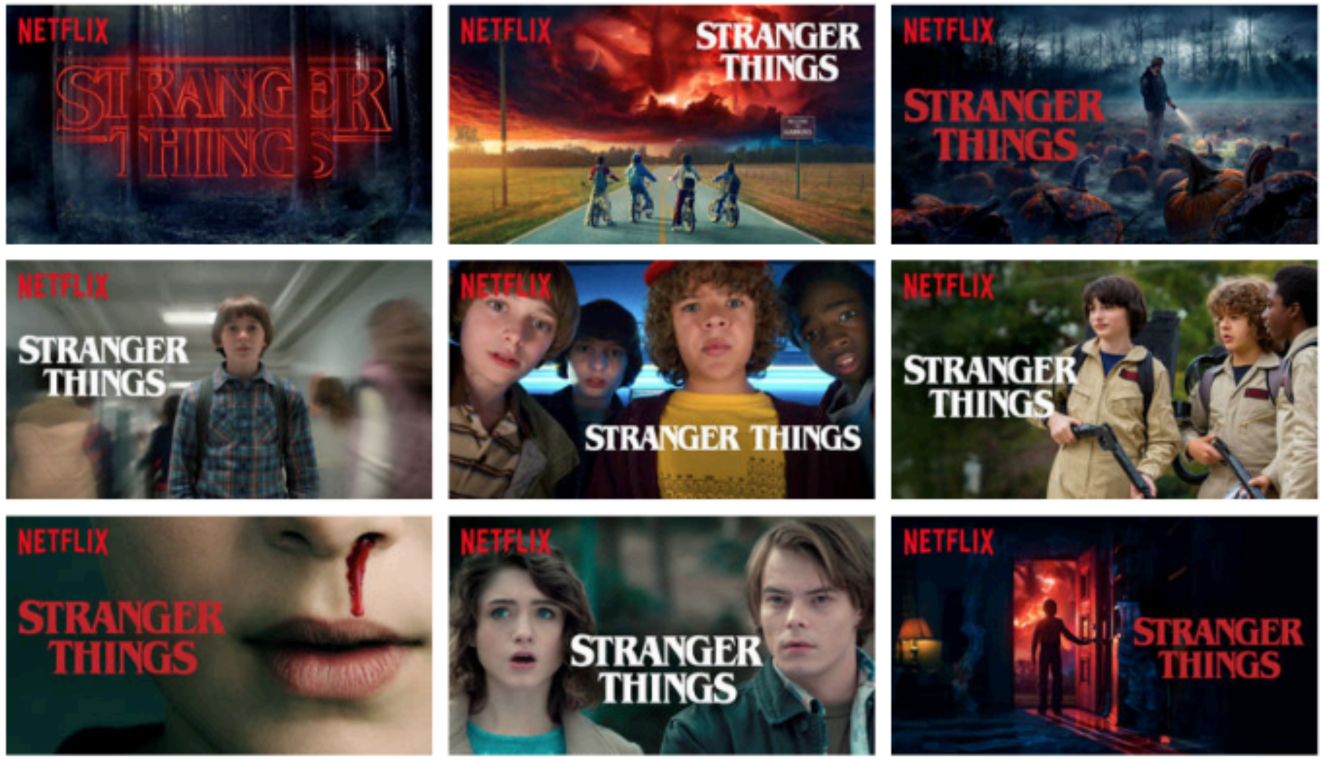


# Real world motivation: NETFLIX artwork

For a particular movie, we want to decide what image to show (to all the NEFLIX users)

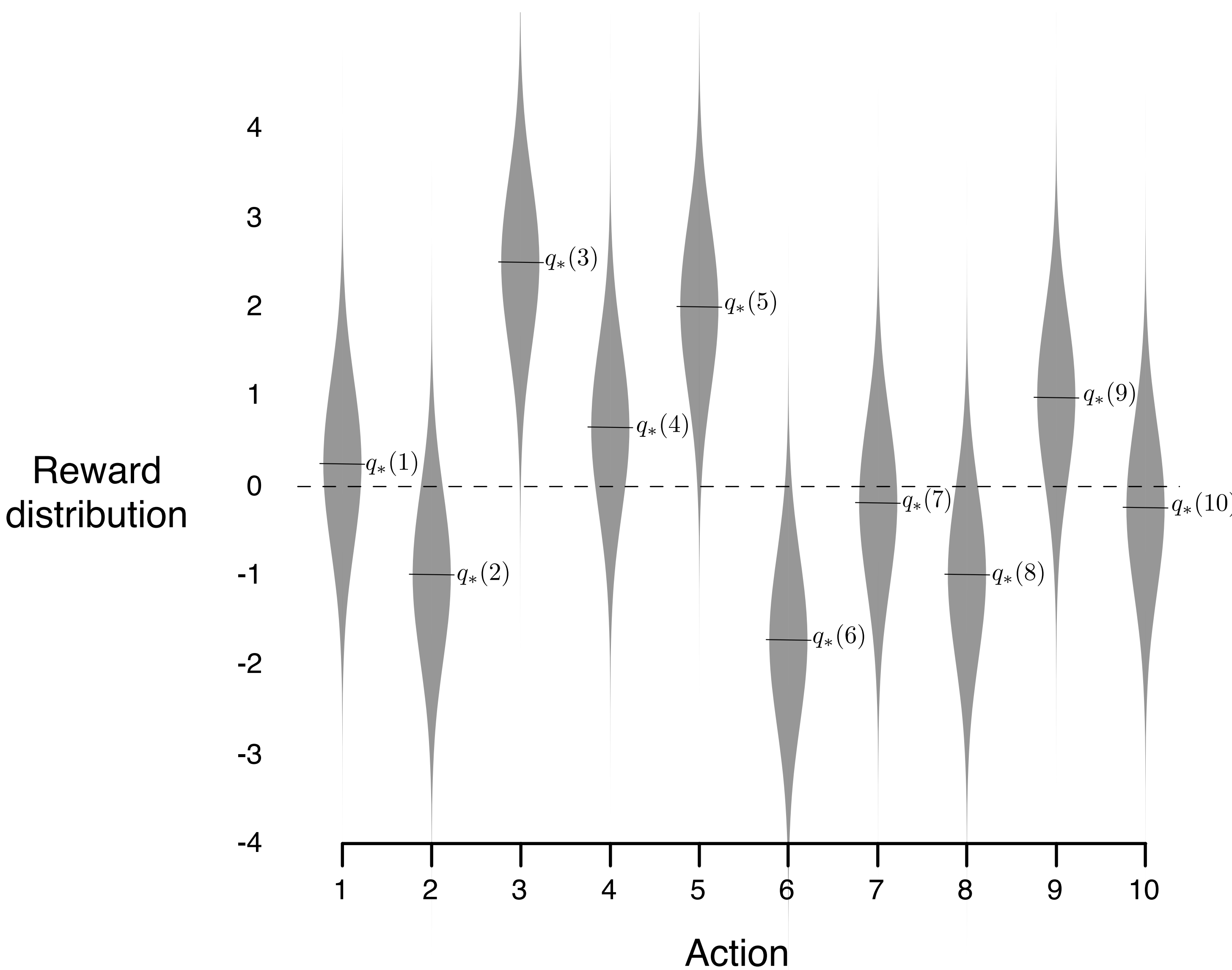
- Actions: uploading one of the K images to a user's home screen
- Reward: 1 if the user clicks and watches, 0 otherwise.
- Mean reward (success probability) for each image: the percentage of users that clicked and watched  (quality engagement, not clickbait)




Netflix Artwork

# Example: Gaussian Bandits

- Each action (arm when played) results in a real number.
- Action (arm) $k \in \{1...K\}$ produces on average reward equal to the mean of its Gaussian distribution.



# Regret

- The action-value is the mean reward for action a,

$$q_*(a) \doteq \mathbb{E}[R_t | A_t = a] , \quad \forall a \in \{1, \ldots, k\}$$

- The optimal value is

$$v_* = q(a^*) = \max_{a \in \mathcal{A}} q_*(a)$$

- The regret is the opportunity loss for one step. For an algorithm that selects action $a_t$ at timestep t it reads:

$$I_t = \mathbb{E}[v_* - q_*(a_t)]$$

reward = – regret

- The total regret is the total opportunity loss

$$L_T = \mathbb{E}\left[\sum_{t=1}^{T} v_* - q_*(a_t)\right]$$

- Maximize cumulative expected reward = minimize total regret

# Regret

- The count $N_t(a)$: the number of times that action a has been selected prior to time $t$
- The gap $\Delta a$ is the difference in value between action $a$ and optimal action $a_*$: $\Delta_a = v_* - q_*(a)$
- Regret is a function of gaps and the counts

$$L_T = \mathbb{E}\left[\sum_{t=1}^{T} v_* - q_*(a_t)\right]$$

$$= \sum_{a \in \mathcal{A}} \mathbb{E}[N_t(a)](v_* - q_*(a))$$

$$= \sum_{a \in \mathcal{A}} \mathbb{E}[N_t(a)]\Delta_a$$

# Forming Action-Value Estimates

- To simplify notation, let us focus on one action
  - We consider only its rewards, and its estimate after n-1 rewards:

$$Q_n \doteq \frac{R_1 + R_2 + \cdots + R_{n-1}}{n-1}$$

- How can we do this incrementally (without storing all the rewards)?
- Could store a running sum and count (and divide), or equivalently:

$$Q_{n+1} = Q_n + \frac{1}{n}\Big[R_n - Q_n\Big]$$

- This is a standard form for learning/update rules:

$$NewEstimate \leftarrow OldEstimate \;+\; StepSize\,\Big[Target - OldEstimate\Big]$$

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error

$$NewEstimate \leftarrow OldEstimate \; + \; StepSize \Big[Target - OldEstimate\Big]$$

# Derivation of incremental update

$$Q_n \doteq \frac{R_1 + R_2 + \cdots + R_{n-1}}{n-1}$$

$$\begin{aligned}
Q_{n+1} &= \frac{1}{n}\sum_{i=1}^{n} R_i \\
&= \frac{1}{n}\left(R_n + \sum_{i=1}^{n-1} R_i\right) \\
&= \frac{1}{n}\left(R_n + (n-1)\frac{1}{n-1}\sum_{i=1}^{n-1} R_i\right) \\
&= \frac{1}{n}\Big(R_n + (n-1)Q_n\Big) \\
&= \frac{1}{n}\Big(R_n + nQ_n - Q_n\Big) \\
&= Q_n + \frac{1}{n}\Big[R_n - Q_n\Big],
\end{aligned}$$

# Non-stationary bandits

- Suppose the true action values change slowly over time
  - then we say that the problem is nonstationary
- In this case, sample averages are not a good idea
  - Why?

# Non-stationary bandits

- Suppose the true action values change slowly over time
  - then we say that the problem is nonstationary
- In this case, sample averages are not a good idea
- Better is an “exponential, recency-weighted average”:

$$
\begin{aligned}
Q_{n+1} &\doteq Q_n + \alpha\Big[R_n - Q_n\Big] \\
&= (1-\alpha)^n Q_1 + \sum_{i=1}^{n} \alpha(1-\alpha)^{n-i} R_i,
\end{aligned}
$$

where $\alpha \in (0,1]$ and constant

The smaller the i, the smaller $(1-a)^{n-i}$-> forgetting earlier rewards

# Action selection in multi-armed bandits

# Fixed exploration period + Greedy

1. Allocate a fixed time period to exploration when you try bandits uniformly at random
2. Estimate mean rewards for all actions: $Q_t(a) = \frac{1}{N_t(a)} \sum_{i=1}^{t-1} r_i \mathbf{1}(A_i = a)$
3. Select the action that is optimal for the estimated mean rewards, breaking ties at random: $a_t = \operatorname{argmax}_{a \in \mathcal{A}} Q_t(a)$
4. GOTO 2

# Fixed exploration period + Greedy

- After the fixed exploration period we have formed the following reward estimates


$Q_t(a_1) = 0.3$


$Q_t(a_2) = 0.5$


$Q_t(a_3) = 0.1$

Q1: Will the greedy method always pick the second action?

Q2: Can greedy lock onto a suboptimal action forever?

⇒ Greedy has linear total regret

# ε-Greedy Action Selection

- In greedy action selection, you always exploit
- In $\varepsilon$-greedy, you are usually greedy, but with probability $\varepsilon$ you instead pick an action at random (possibly the greedy action again)
- This is perhaps the simplest way to balance exploration and exploitation

# ε-Greedy Action Selection

**A simple bandit algorithm**

Initialize, for $a = 1$ to $k$:

$Q(a) \leftarrow 0$

$N(a) \leftarrow 0$

Repeat forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

$R \leftarrow bandit(A)$

$N(A) \leftarrow N(A) + 1$

$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\big[R - Q(A)\big]$

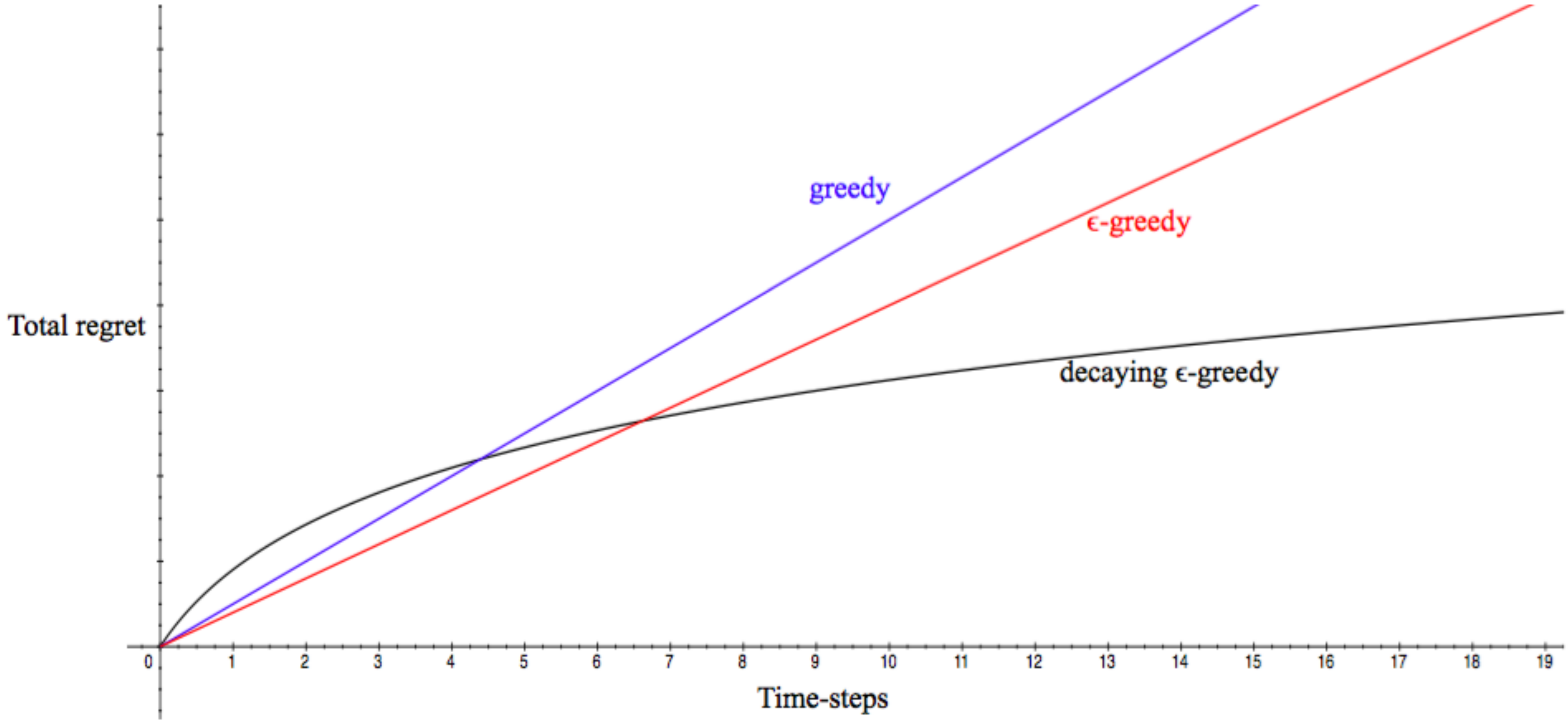
# ε-Greedy Algorithm

- The ε-greedy algorithm continues to explore forever
  - With probability 1 – ε select $a_t = \text{argmax}_{a \in \mathcal{A}} Q_t(a)$
  - With probability ε select a random action (independent of its Q estimate)
- Constant ε ensures minimum regret

$$I_t \geq \frac{\epsilon}{|\mathcal{A}|} \sum_{a \in \mathcal{A}} \Delta_a$$

- ⇒ ε-greedy has linear total regret

# Counting Regret

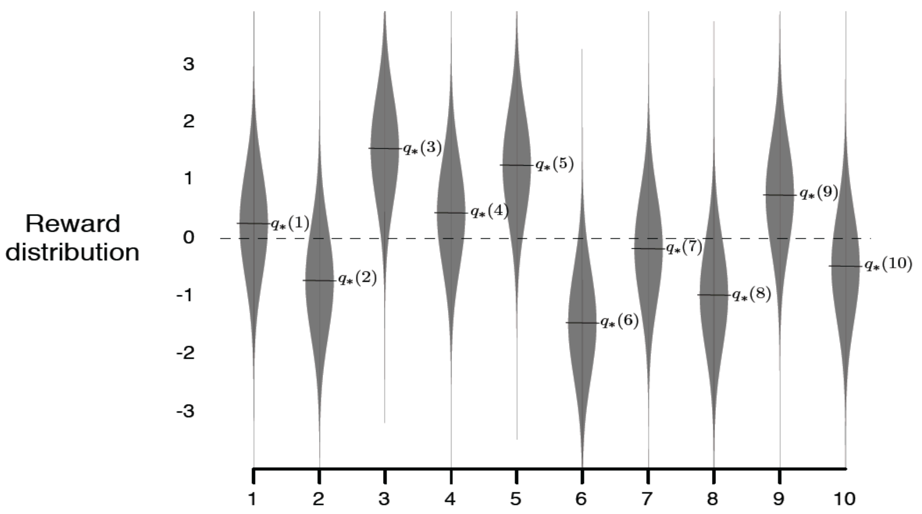



- If an algorithm forever explores it will have linear total regret
- If an algorithm never explores it will have linear total regret

# Average reward for three algorithms

We sample 10 arm bandits instantiations:

$$q_*(a) \sim \mathcal{N}(0,1)$$
$$R_t \sim \mathcal{N}(q_*(a),1)$$



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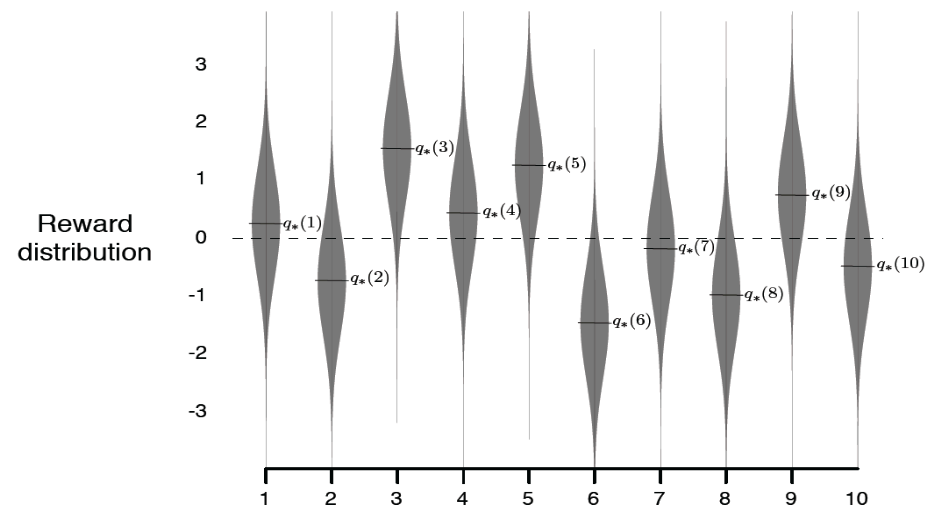


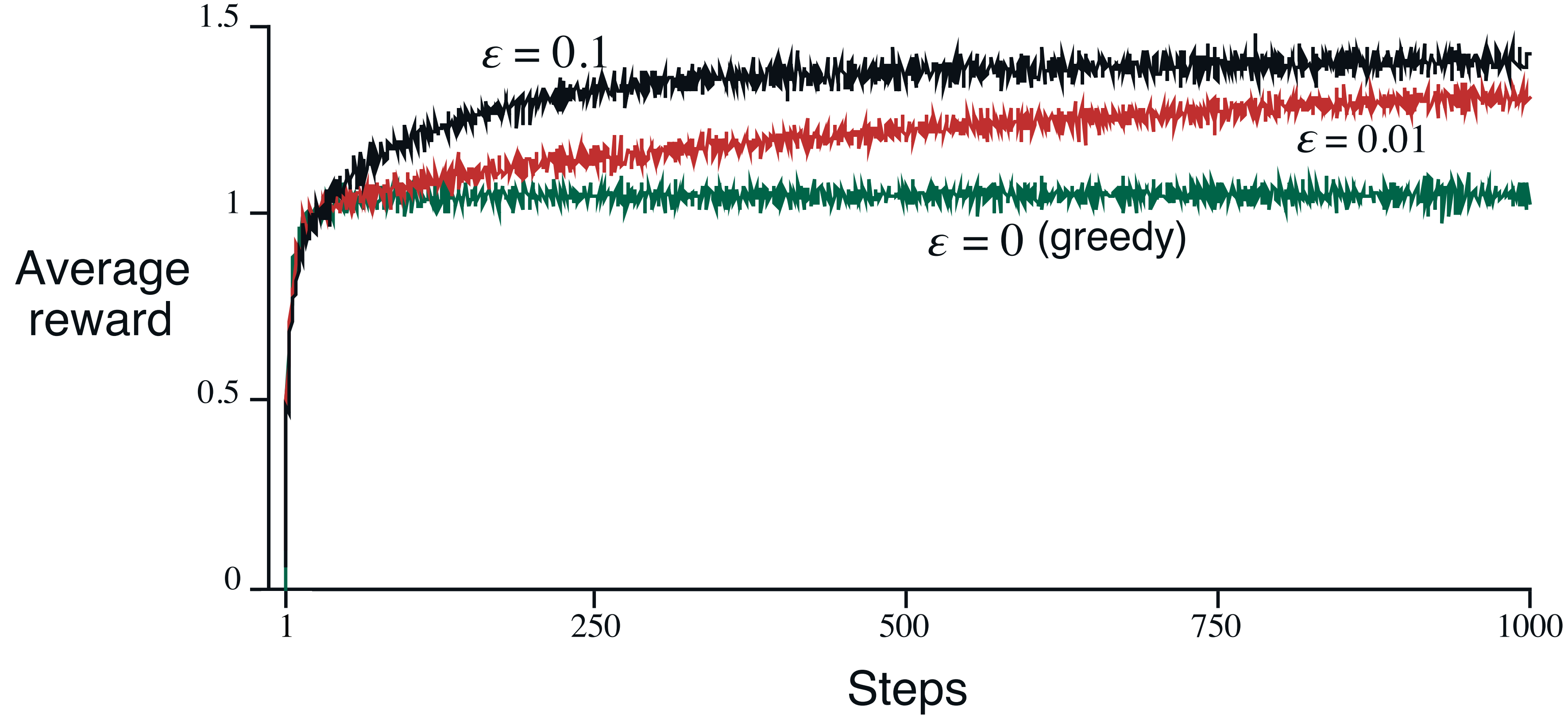


Q: In the limit (after infinite number of steps), which method will result in the largest average reward?

# Optimal action for three algorithms

We sample 10 arm bandits instantiations:

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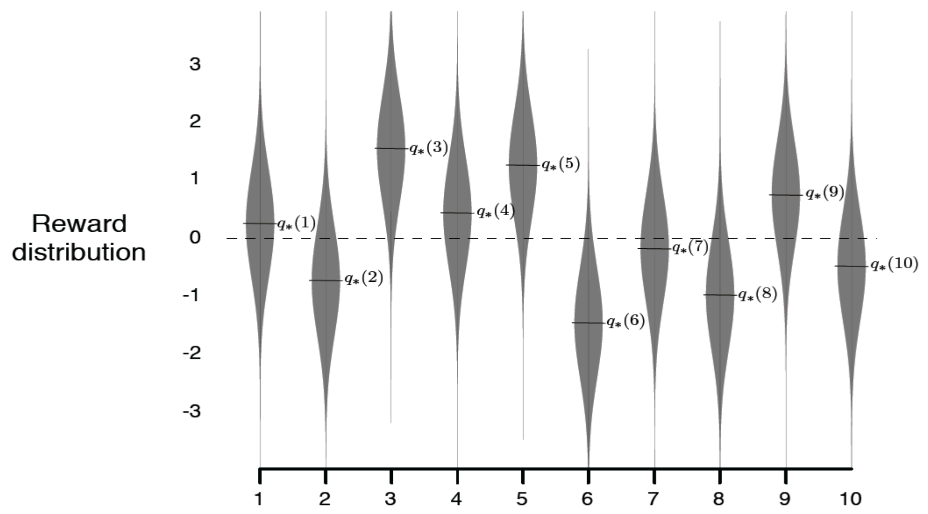


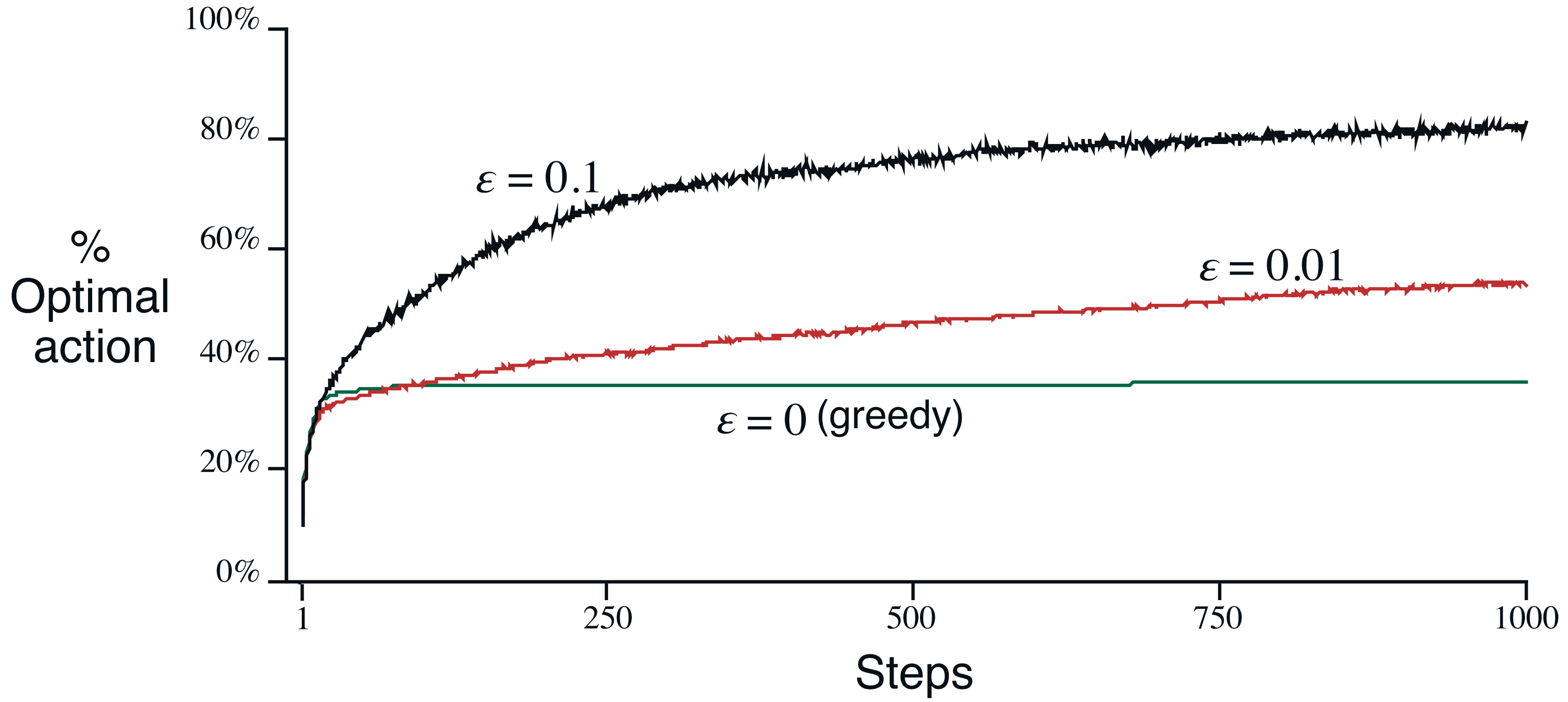


Q: Which method will find the optimal action in the limit?

# Optimal action for three algorithms

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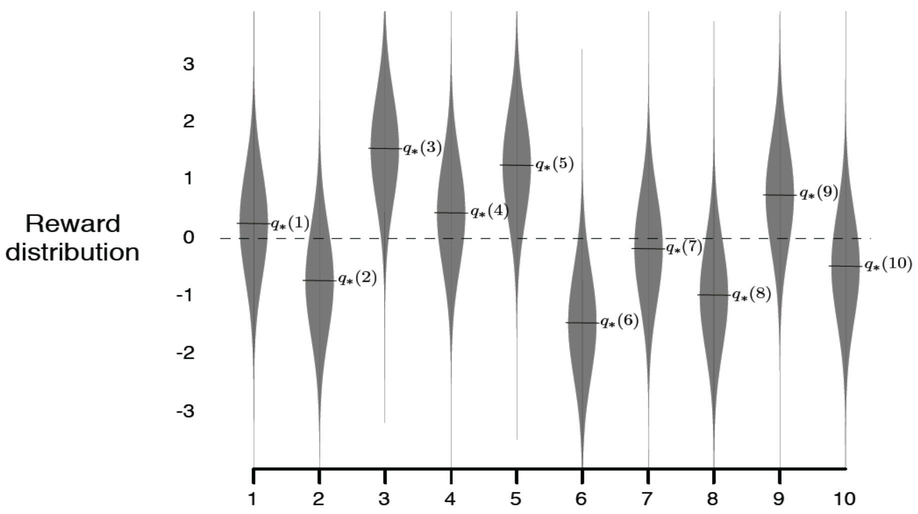
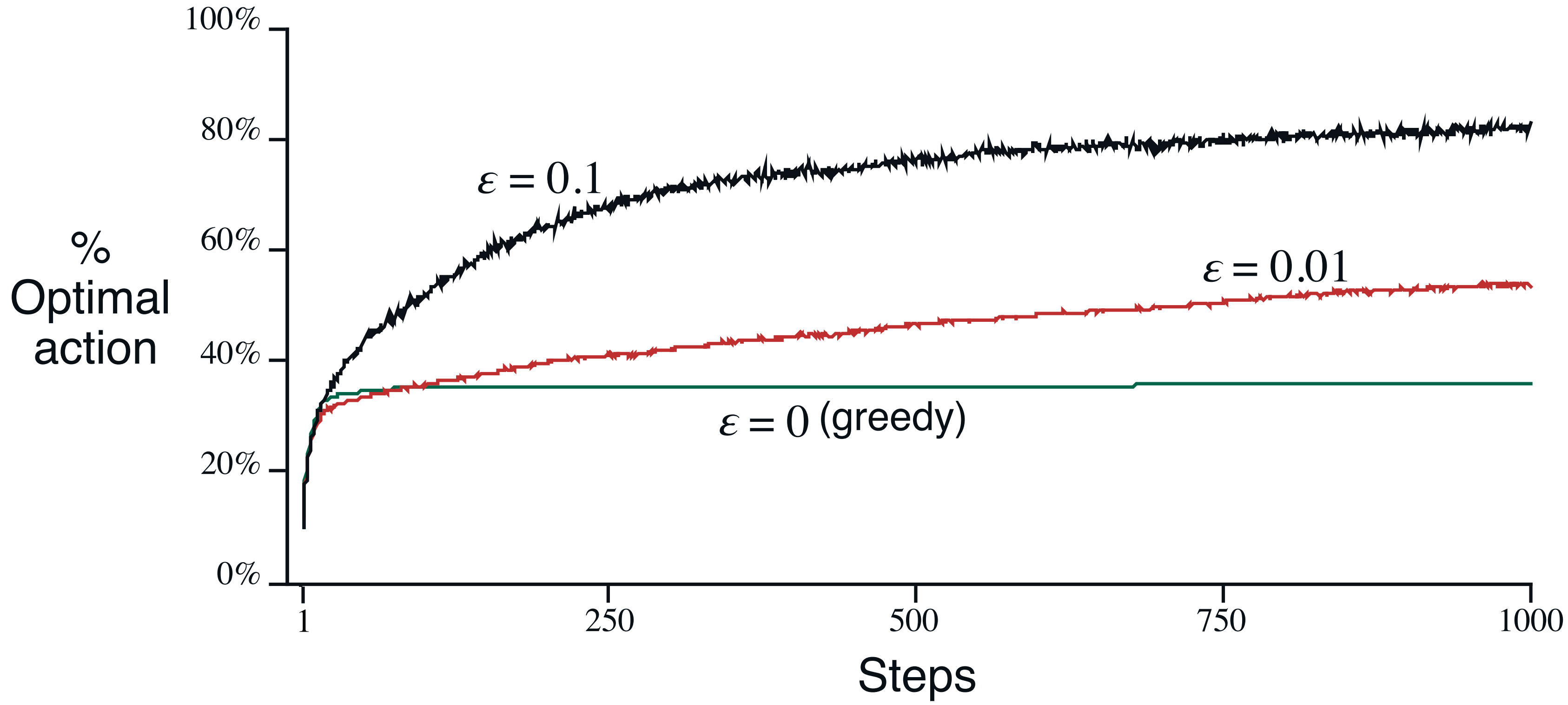



Q: Does the performance of those methods depend on the initialization of the action value estimates?

# Optimistic Initialization

- Simple and practical ideas: initialise $Q(a)$ to a high value
- Update action value by incremental Monte-Carlo evaluation
- Starting with $N(a) > 0$,

$$Q_t(a_t) = Q_{t-1}(a_t) + \frac{1}{N_t(a_t)}\left(r_t - Q_{t-1}(a_t)\right)$$

just an incremental estimate of sample mean, including one 'hallucinated' initial optimistic value

- Encourages systematic exploration early on
- But optimistic greedy can still lock onto a suboptimal action if rewards are stochastic.

# Optimistic Initial Values

We initialize with the following reward estimates for Bernoulli bandits


$Q_t(a_1) = 1$


$Q_t(a_2) = 1$

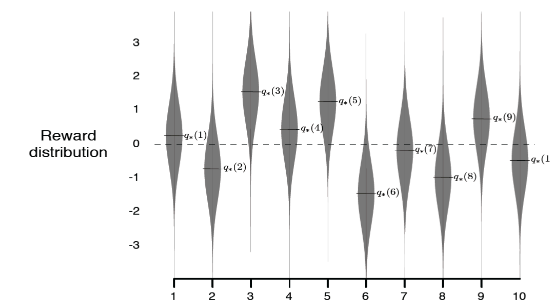

$Q_t(a_3) = 1$

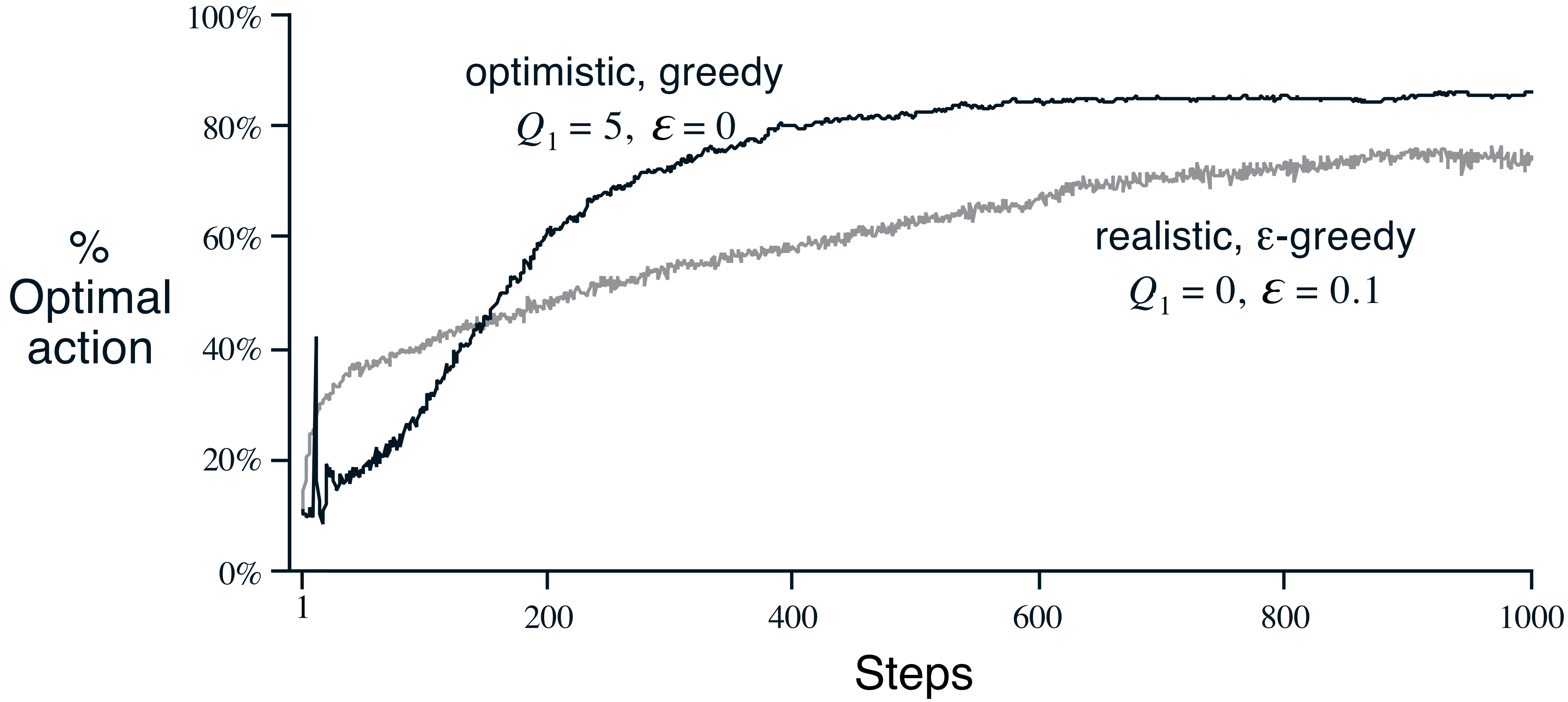
Q: When it is possible that greedy action selection will not try out all the actions?

# Optimistic Initial Values

- Suppose we initialize the action values optimistically ($Q_1(a) = 5$), e.g., on the 10-armed testbed

$$q_*(a) \sim \mathcal{N}(0,1)$$
$$R_t \sim \mathcal{N}(q_*(a),1)$$



# Achieving sub-linear total regret

We need to reason about uncertainty of our action value estimates

1000 pulls,
600 wins
Q_t(a_1)=0.6

1000 pulls,
400 wins
Q_t(a_2)=0.4

10 pulls,
4 wins
Q_t(a_1)=0.4

Epsilon-greedy

Repeat forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

$$R \leftarrow bandit(A)$$

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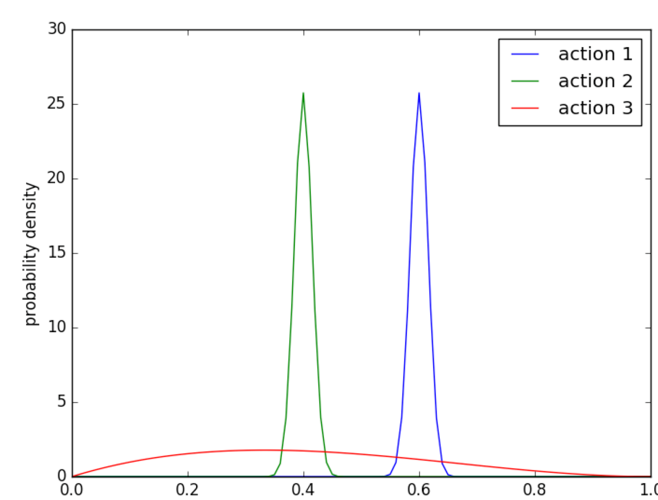
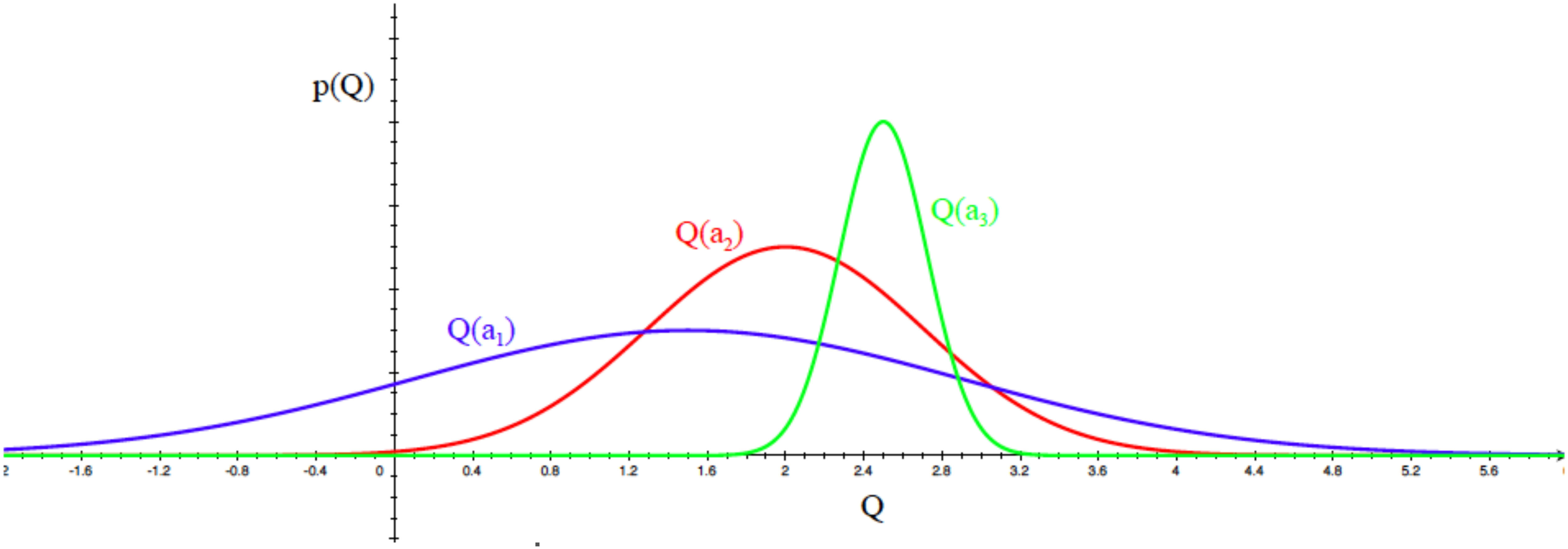
The problem with using mean estimates is that we do not reason about uncertainty of those estimates.

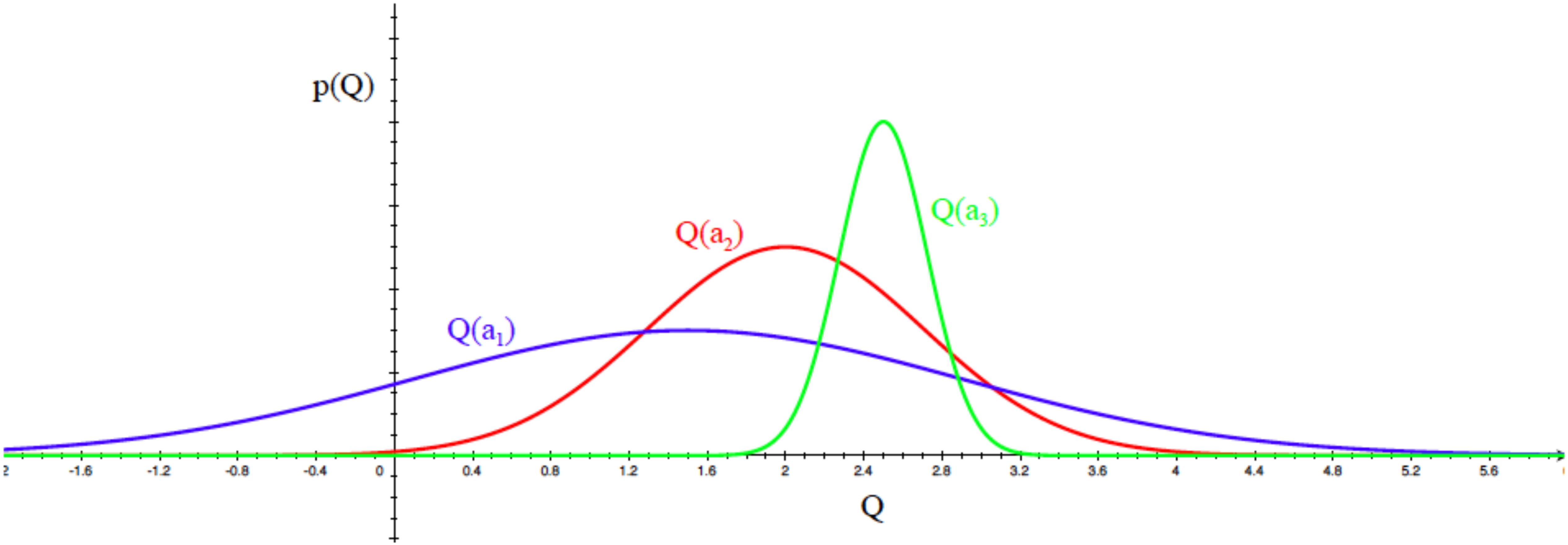


# Uncertainty guides Exploration




- The more uncertain we are about an action-value
- The more important it is to explore that action
- It could turn out to be the best action

# Uncertainty guides Exploration




- We are then less uncertain about the value
- And more likely to pick another action
- Until we converge to the best action

# Upper Confidence Bounds

- Estimate an upper confidence $U_t(a)$ for each action value
- Such that with high probability

$$q_*(a) \leq Q_t(a) + U_t(a)$$

Estimated mean

Estimated Upper Confidence

- This upper confidence depends on the number of times action a has been selected
  - Small $N_t(a) \Rightarrow$ large $U_t(a)$ (estimated value is uncertain)
  - Large $N_t(a) \Rightarrow$ small $U_t(a)$ (estimated value is accurate)
- Select action maximizing Upper Confidence Bound (UCB)

$$a_t = \operatorname{argmax}_{a \in \mathcal{A}} Q_t(a) + U_t(a)$$

# Hoeffding’s Inequality

Let $X_1, \ldots X_t$ be independent random variables in the range [0,1] with $\mathbb{E}(X_i) = \mu$. Then for $u > 0$,

$$\mathbb{P}\left(\frac{1}{n}\sum_{i=1}^{n} X_i \geq \mu + u\right) \leq e^{-2u^2 n}$$

sample mean

$$\mathbb{P}\left(\frac{1}{n}\sum_{i=1}^{n} X_i \leq \mu - u\right) \leq e^{-2u^2 n}$$

Hoeffding's inequality provides an upper bound on the probability that the sum of bounded independent random variables deviates from its expected value by more than a certain amount.

# Hoeffding's Inequality

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$$\mathbb{P}\left(\frac{1}{n}\sum_{i=1}^{n} X_i \geq \mu + u\right) \leq e^{-2u^2 n}$$

I made the margin to depend on the amount of interactions t

- We will apply Hoeffding's Inequality to the rewards obtained from each action (bandit) a:

$$\mathbb{P}\left(\hat{Q}_t(a) \geq q(a) + U_t(a)\right) \leq e^{-2U_t(a)^2 N_t(a)}$$

  - t: how many times I have played any action,
  - $N_t(a)$: how many times I have played action a in t interactions

# Calculating Upper Confidence Bounds

- Pick a probability $p$ that the value estimate deviates from its mean
- Now solve for $U_t(a)$

$$e^{-2U_t(a)^2 N_t(a)} = p$$

$$U_t(a) = \sqrt{\frac{-\log p}{2N_t(a)}}$$

- Reduce p as we play more, e.g. $p = t^{-c}, c = 4$
- Ensures we select optimal action as t → ∞

$$U_t(a) = \sqrt{\frac{2 \log t}{2N_t(a)}}$$

# Upper Confidence Bound (UCB)

- A clever way of reducing exploration over time
- Estimate an upper bound on the true action values
- Select the action with the largest (estimated) upper bound

$$A_t \doteq \arg\max_a \left[ Q_t(a) + c\sqrt{\frac{\log t}{N_t(a)}} \right]$$

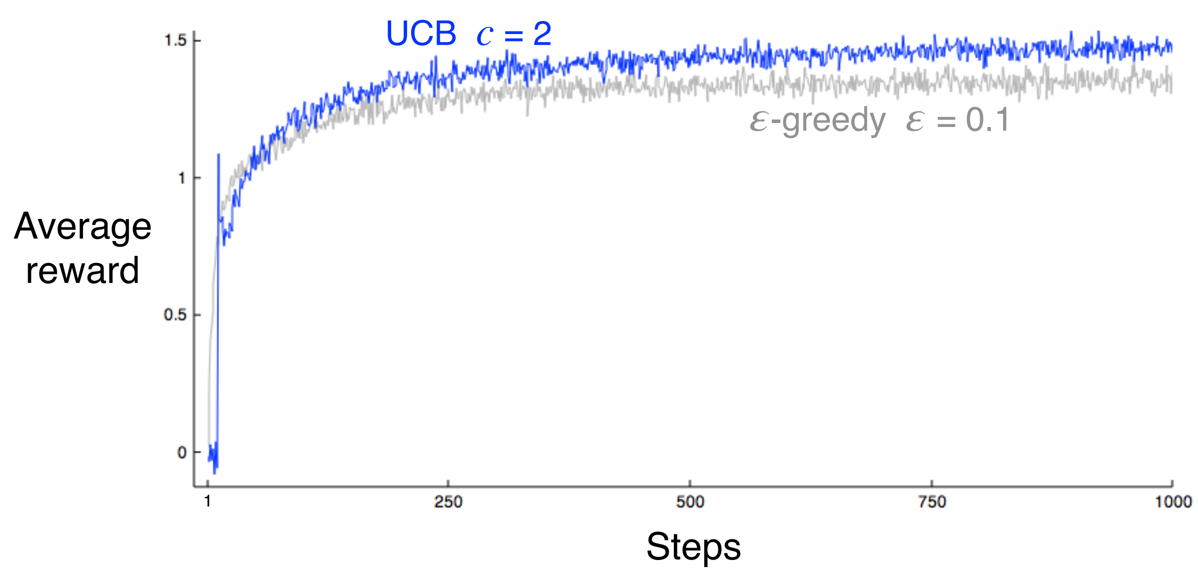
- c is a hyper-parameter that trades-off explore/exploit
- the confidence bound grows with the total number of actions we have taken t but shrinks with the number of times we have tried this particular action $N_t(a)$. This ensures each action is tried infinitely often but still balances exploration and exploitation.

UCB1: Auer, Cesa-bianchi, and Fischer, Finite-time analysis of the multiarmed bandit problem, 2002

# Upper Confidence Bound (UCB)

- A clever way of reducing exploration over time
- Estimate an upper bound on the true action values
- Select the action with the largest (estimated) upper bound

$$A_t \doteq \underset{a}{\arg\max} \left[ Q_t(a) + c\sqrt{\frac{\log t}{N_t(a)}} \right]$$




UCB1: Auer, Cesa-bianchi, and Fischer, Finite-time analysis of the multiarmed bandit problem, 2002

# UCB1 Algorithm

- This leads to the UCB1 algorithm

$$a_t = \operatorname*{argmax}_{a \in \mathcal{A}} Q(a) + \sqrt{\frac{2 \log t}{N_t(a)}}$$

**Theorem**

*The UCB algorithm achieves logarithmic asymptotic total regret*

$$\lim_{t \to \infty} L_t \leq 8 \log t \sum_{a | \Delta_a > 0} \Delta_a$$

UCB1: Auer, Cesa-bianchi, and Fischer, Finite-time analysis of the multiarmed bandit problem, 2002

# Bayesian Bandits

- So far we have made no assumptions about the reward distributions.
  - In UCB we just considered some bounds on rewards
- Bayesian bandits exploit prior knowledge of rewards, $p[\mathcal{R}]$
- They compute posterior distribution of rewards $p[\mathcal{R} \mid h_t]$

$$h_t = a_1, r_1, ..., a_{t-1}, r_{t-1}$$

- Use posterior to guide exploration: we simply sample from the posterior!

# Bayesian learning for model parameters

Step 1: Given $n$ data, $D = x_{1...n} = \{x_1, x_2, \ldots, x_n\}$ write down the expression for likelihood:

$$p(D \mid \theta)$$

Step 2: Specify a prior: $p(\theta)$

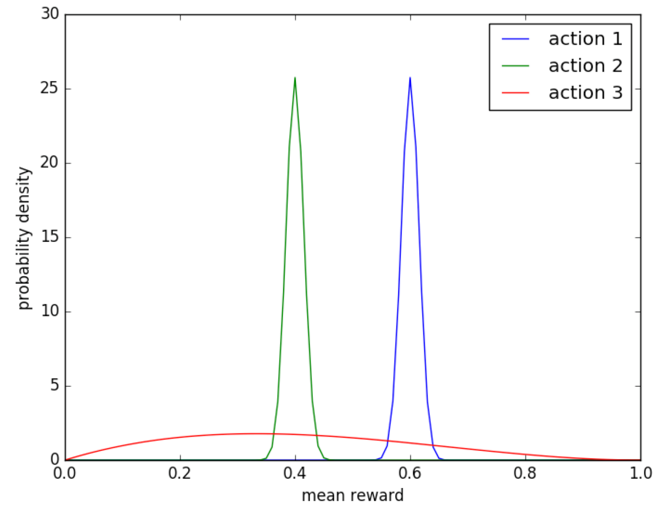
Step 3: Compute the posterior:

$$p(\theta \mid D) = \frac{p(D \mid \theta) p(\theta)}{p(D)}$$

# Thompson Sampling

Represent a distribution for the mean reward of each bandit as opposed to the mean reward estimate alone. At each timestep:

1. Sample from the mean reward distributions:
   $\bar{\theta}_1 \sim \hat{p}(\theta_1), \bar{\theta}_2 \sim \hat{p}(\theta_2), \cdots, \bar{\theta}_k \sim \hat{p}(\theta_k)$




2. Choose action $a = \arg\max_a \mathbb{E}_{\bar{\theta}}[r(a)]$

3. Observe the reward.

4. Update the mean reward posterior distributions: $\hat{p}(\theta_1), \hat{p}(\theta_2) \cdots \hat{p}(\theta_k)$

Q: why we use argmax in step 2 and we do not add any noise?

# Bernoulli bandits - Prior

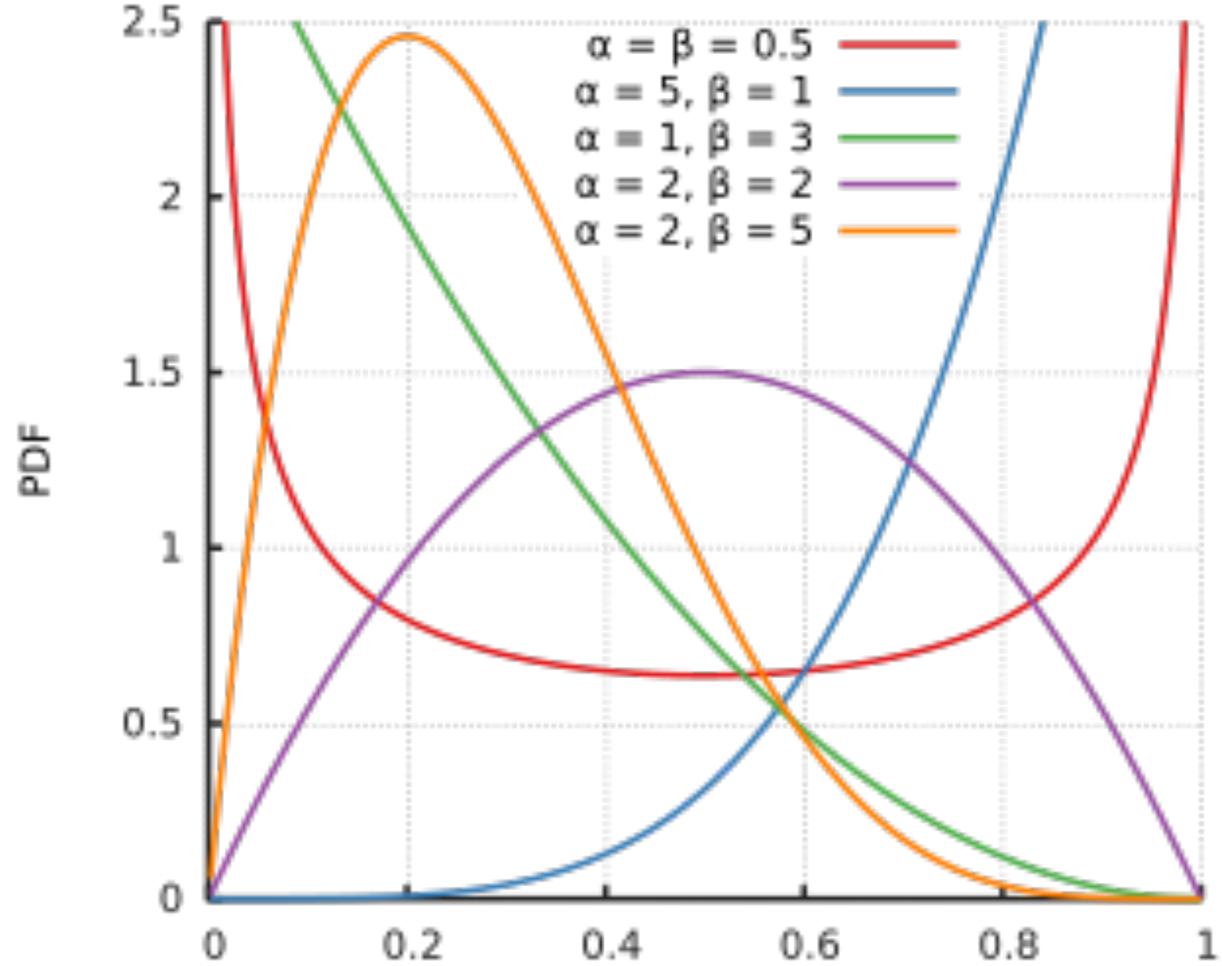
Let's consider a Beta distribution prior over the mean rewards of the Bernoulli bandits:

$$p(\theta_k) = \frac{\Gamma(\alpha_k + \beta_k)}{\Gamma(\alpha_k)\Gamma(\beta_k)} \theta_k^{\alpha_k - 1}(1 - \theta_k)^{\beta_k - 1} \qquad \Gamma(n) = (n-1)!$$

The mean is $\dfrac{\alpha}{\alpha + \beta}$

The larger the $\alpha + \beta$ the more concentrated the distribution

$\mathrm{Beta}(\alpha, \beta)$



# Bernoulli bandits-Posterior

Let's consider a Beta distribution prior over the mean rewards of the Bernoulli bandits:

$$p(\theta_k) = \frac{\Gamma(\alpha_k + \beta_k)}{\Gamma(\alpha_k)\Gamma(\beta_k)} \theta_k^{\alpha_k - 1}(1 - \theta_k)^{\beta_k - 1} \qquad \Gamma(n) = (n-1)!$$

The posterior is also a Beta. Because beta is conjugate distribution for the Bernoulli distribution.

A closed form solution for the bayesian update, possible only for conjugate distributions.

$$(\alpha_k, \beta_k) \leftarrow \begin{cases} (\alpha_k, \beta_k) & \text{if } x_t \neq k \\ (\alpha_k, \beta_k) + (r_t, 1 - r_t) & \text{if } x_t = k. \end{cases}$$

# Greedy VS Thompson for Bernoulli bandits

a: success
b: failure

**Algorithm 1** BernGreedy$(K, \alpha, \beta)$

**for** $t = 1, 2, \ldots$ **do**
  #estimate model:
  **for** $k = 1, \ldots, K$ **do**
    $\hat{\theta}_k \leftarrow \alpha_k / (\alpha_k + \beta_k)$
  **end for**

  #select and apply action:
  $x_t \leftarrow \text{argmax}_k \hat{\theta}_k$
  Apply $x_t$ and observe $r_t$

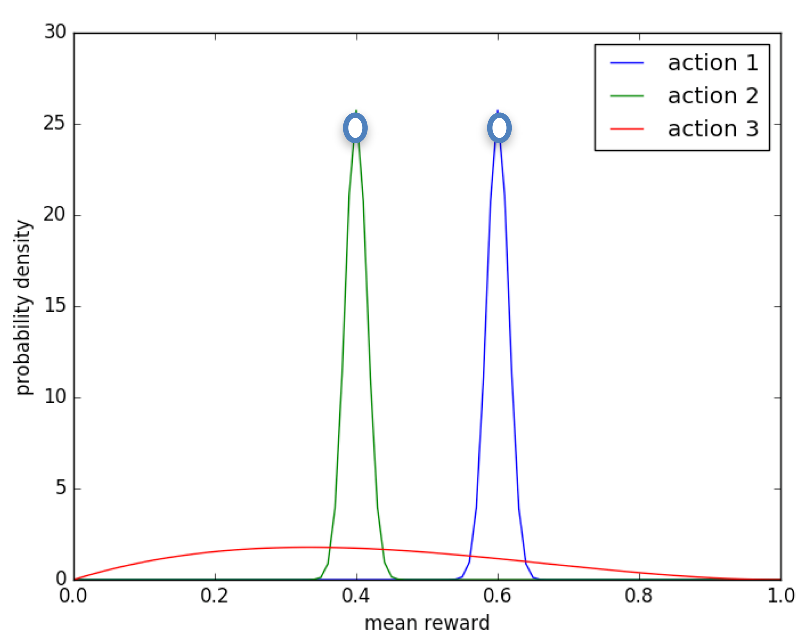
  #update distribution:
  $(\alpha_{x_t}, \beta_{x_t}) \leftarrow (\alpha_{x_t}, \beta_{x_t}) + (r_t, 1 - r_t)$
**end for**

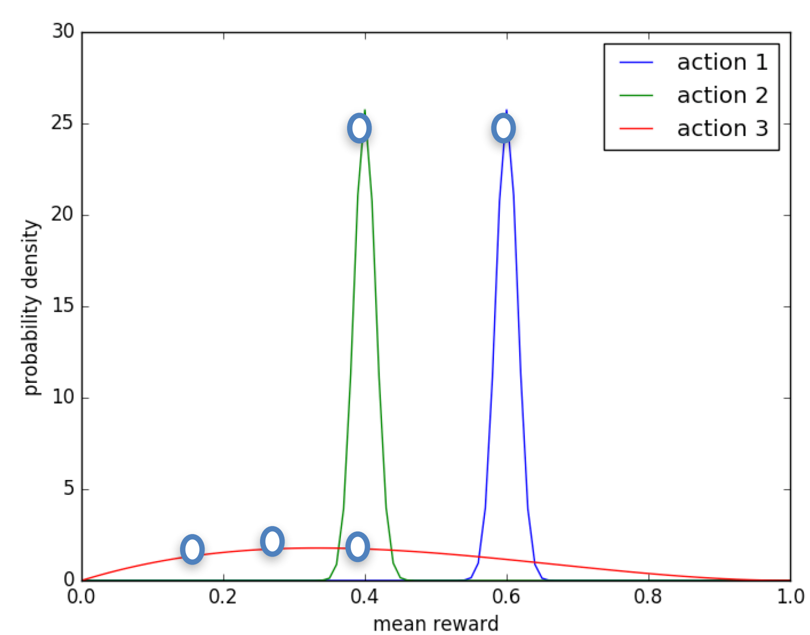
**Algorithm 2** BernThompson$(K, \alpha, \beta)$

**for** $t = 1, 2, \ldots$ **do**
  #sample model:
  **for** $k = 1, \ldots, K$ **do**
    Sample $\hat{\theta}_k \sim \text{beta}(\alpha_k, \beta_k)$
  **end for**

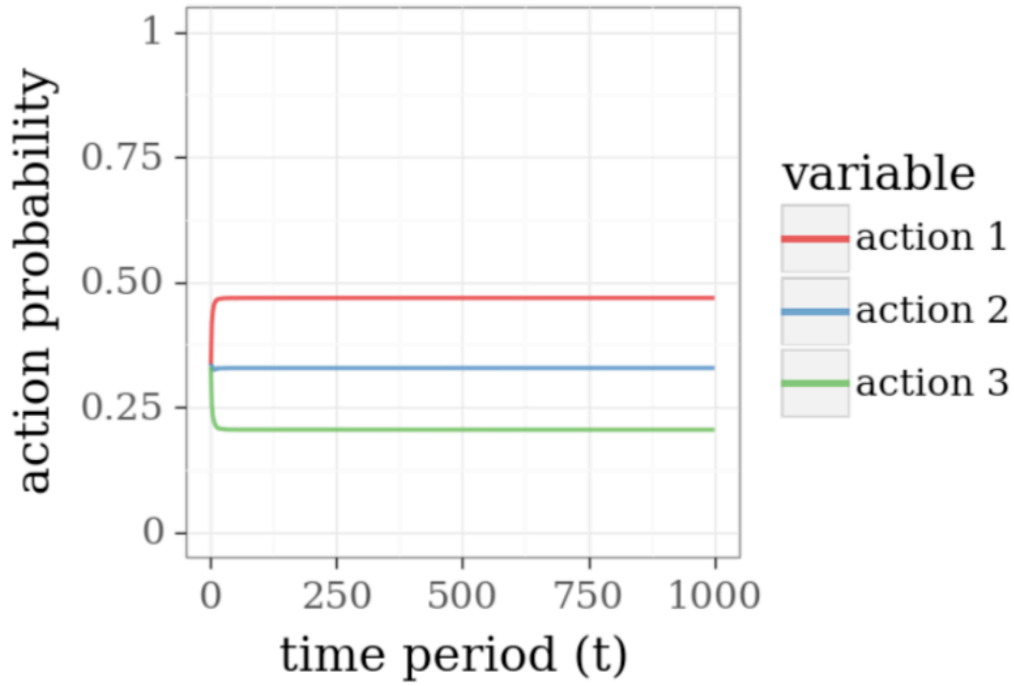
  #select and apply action:
  $x_t \leftarrow \text{argmax}_k \hat{\theta}_k$
  Apply $x_t$ and observe $r_t$

  #update distribution:
  $(\alpha_{x_t}, \beta_{x_t}) \leftarrow (\alpha_{x_t}, \beta_{x_t}) + (r_t, 1 - r_t)$
**end for**

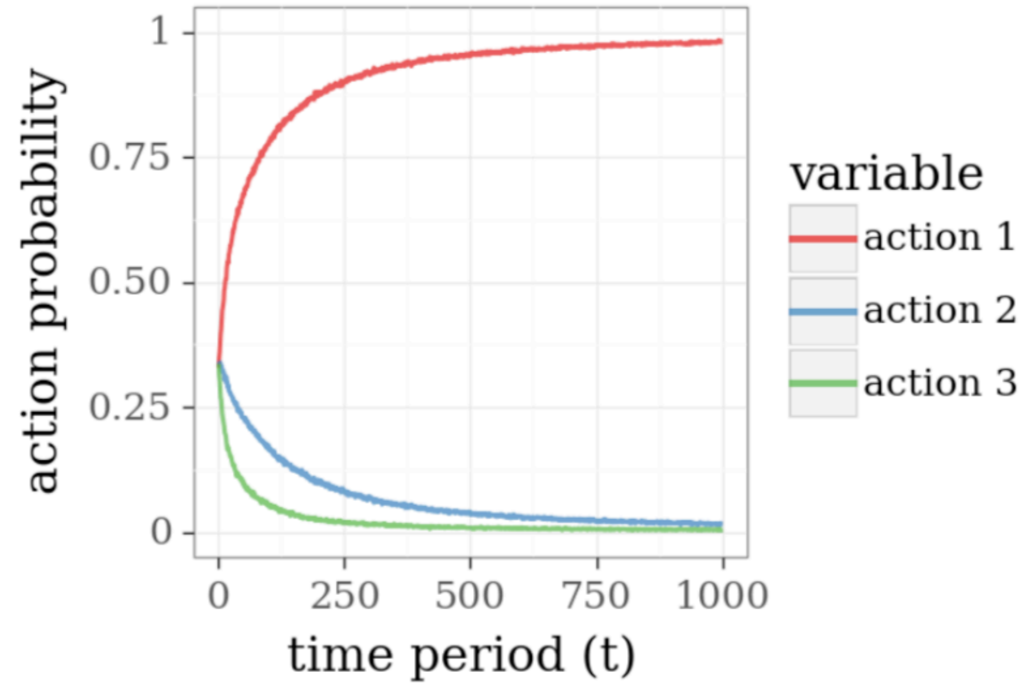


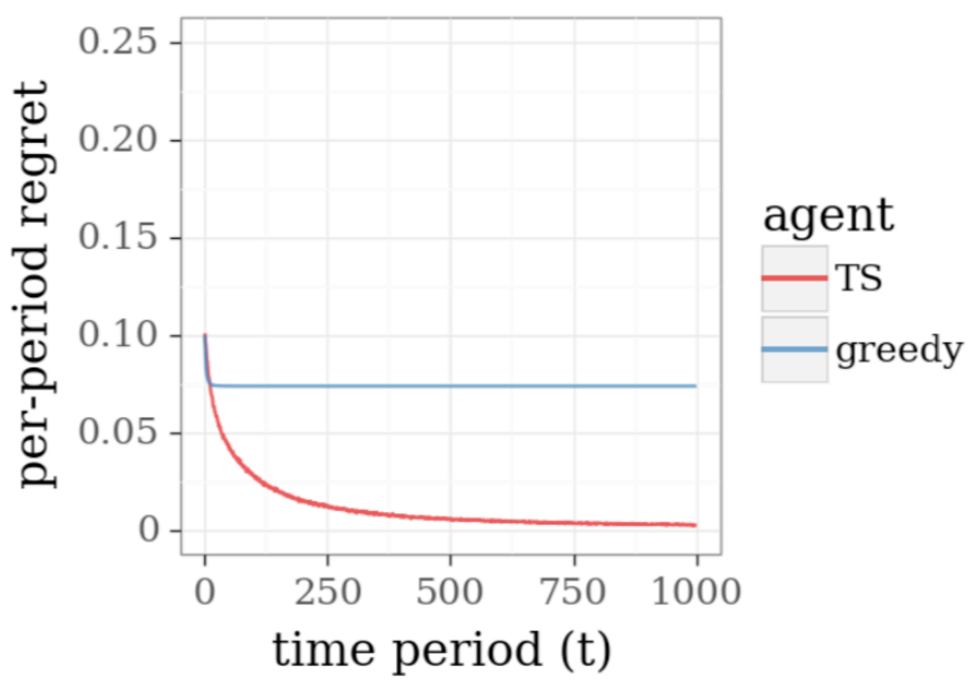
Using uniform prior in [0,1] for the success probabilities



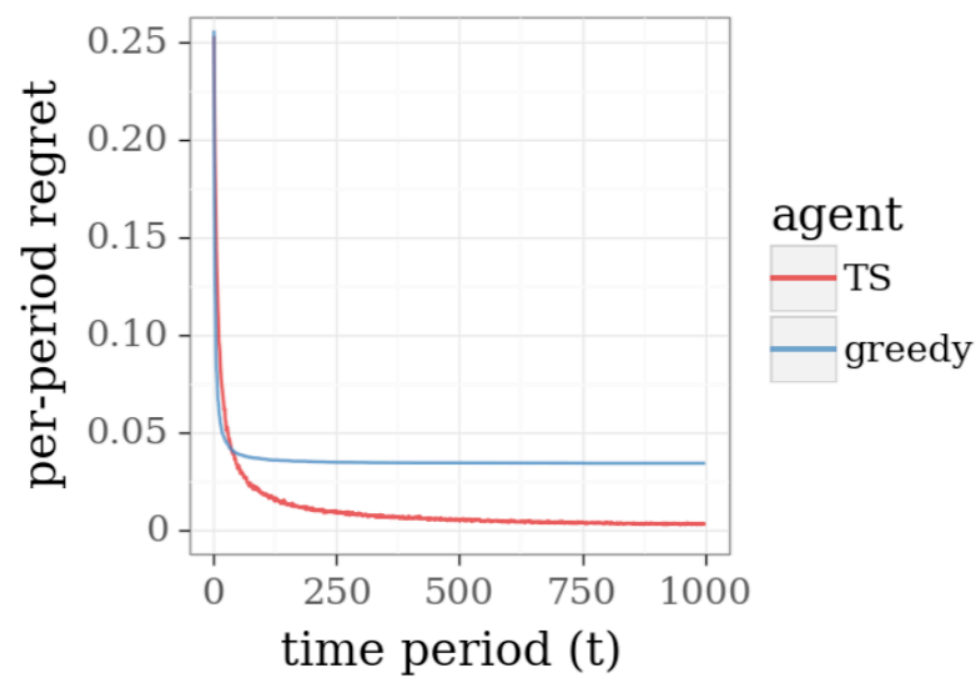

**(a)** greedy algorithm




**(b)** Thompson sampling




**(a)** $\theta = (0.9, 0.8, 0.7)$




**(b)** average over random $\theta$

# Contextual Bandits (a.k.a Associative Search)

- A contextual bandit is a tuple $(A, S, R)$
- A is a known set of k actions (or "arms")
- $\mathcal{S} = \mathbb{P}[s]$ is an unknown distribution over states (or "contexts")
- $\mathcal{R}_s^a(r) = \mathbb{P}[r \mid s, a]$ is an unknown probability distribution over rewards
- At each time $t$
  - Environment generates state $s_t \sim \mathcal{S}$
  - Agent selects action $a_t \in \mathcal{A}$
  - Environment generates reward $r_t \sim \mathcal{R}_{s_t}^{a_t}$
- The goal is to maximize cumulative reward $\sum_{\tau=1}^{t} r_\tau$

# Real world motivation: Personalized NETFLIX artwork



For a particular title and a particular user, we will use the contextual multi-armed bandit formulation to decide what image to show per title per user

- Actions: uploading an image (available for this movie title) to a user's home screen
- Mean rewards (unknown): the % of NETFLIX users that will click on the title and watch the movie
- Estimated mean rewards: the average click rate (+quality engagement, not clickbait)
- Context (s) : user attributes, e.g., language preferences, gender of films she has watched, time and day of the week, etc.

Netflix Artwork:https://medium.com/netflix-techblog/artwork-personalization-c589f074ad76

# Question: was there a train and test phase on our multi-armed bandit algorithms?

No, the setup we explored was: given a set of K arms, how do we select actions to minimize our cumulative regret.

Q: what would be the learning based equivalent of the multi-armed bandit problem?

A:

- We have a training set of N multi-armed bandit instantiations.
- Each K-armed bandit is one training example.
- The agent gets n number of interactions, and obtains a final reward (- regret).
- The agent learns a policy —mapping from its set of actions taken thus far and their outcomes, to a probability over what actions to try next