Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Learning from RL and demonstations, Adversarial imitation learning,

Fall 2021, CMU 10-703

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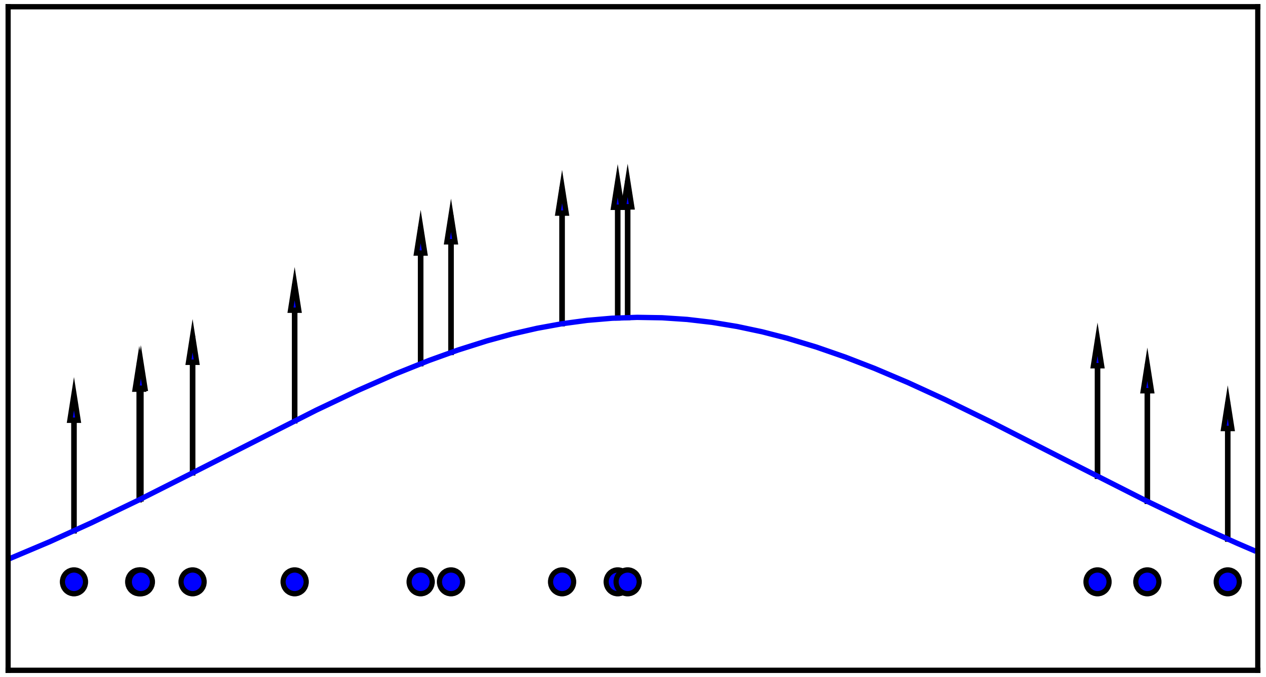
Last lecture

- Behaviour cloning for imitation learning. Assumes access to a set of trajectories $\mathcal{T} = \{o_1^j, a_1^j, o_2^j, a_2^j, o_3^j, a_3^j, \ldots, o_T^j, a_T^j, j = 1...T\}$. Trains a policy by minimizing a standard supervised learning objective:

$$\mathcal{L}_{BC}(\theta, \mathcal{T}) = \mathbb{E}_{(s_t^j, a_t^j) \sim \mathcal{T}} \left[\|a_t^j - \pi_\theta(s_t^j)\|_2^2 \right]$$

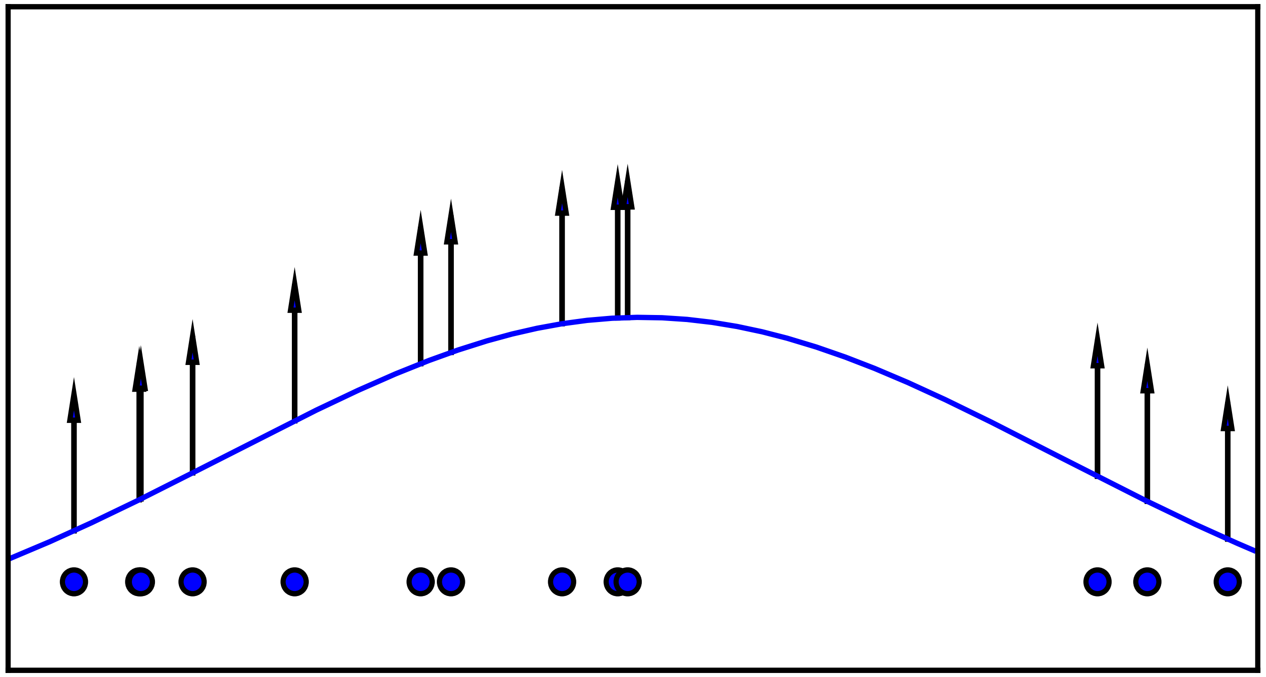
Maximum Likelihood



$$\theta^* = \arg\max_\theta \mathbb{E}_{x\sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta)$$

$$\theta^* = \arg\max_\theta \sum_{i=1}^{N} \log p_{\text{model}}(\mathbf{x}_i \mid \theta)$$

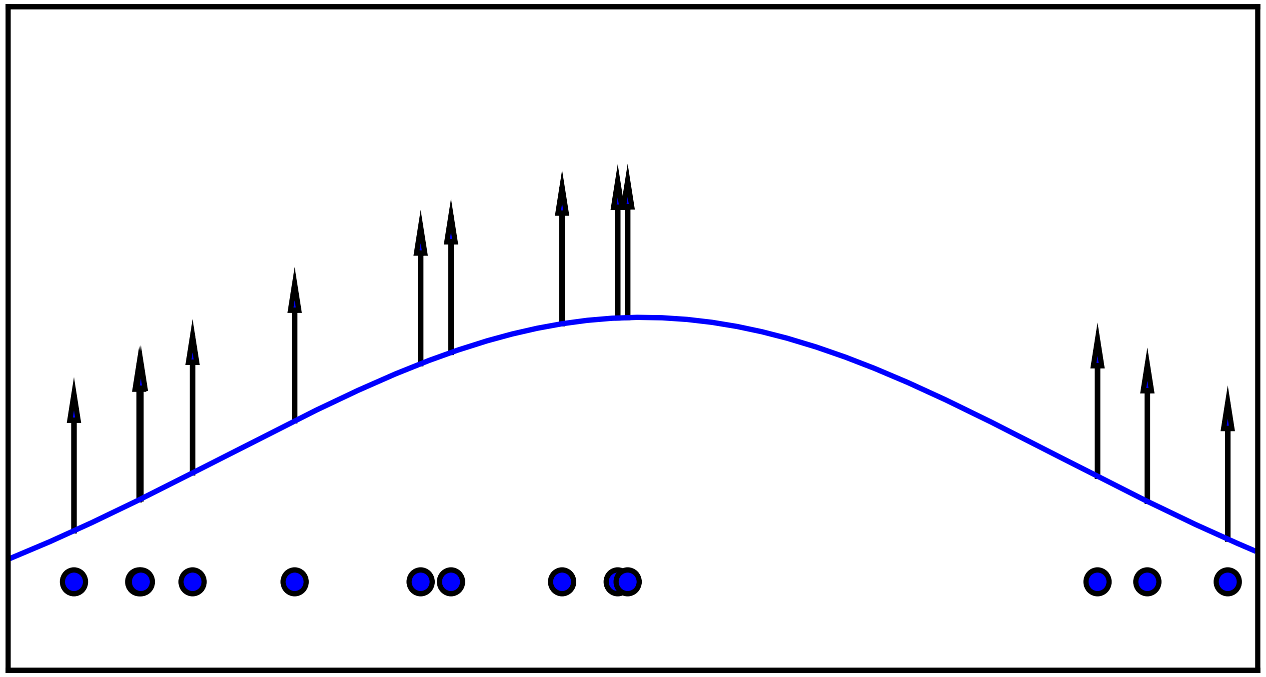
Maximum Likelihood



$$\theta^* = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta)$$

explicit density

Maximum Conditional Likelihood



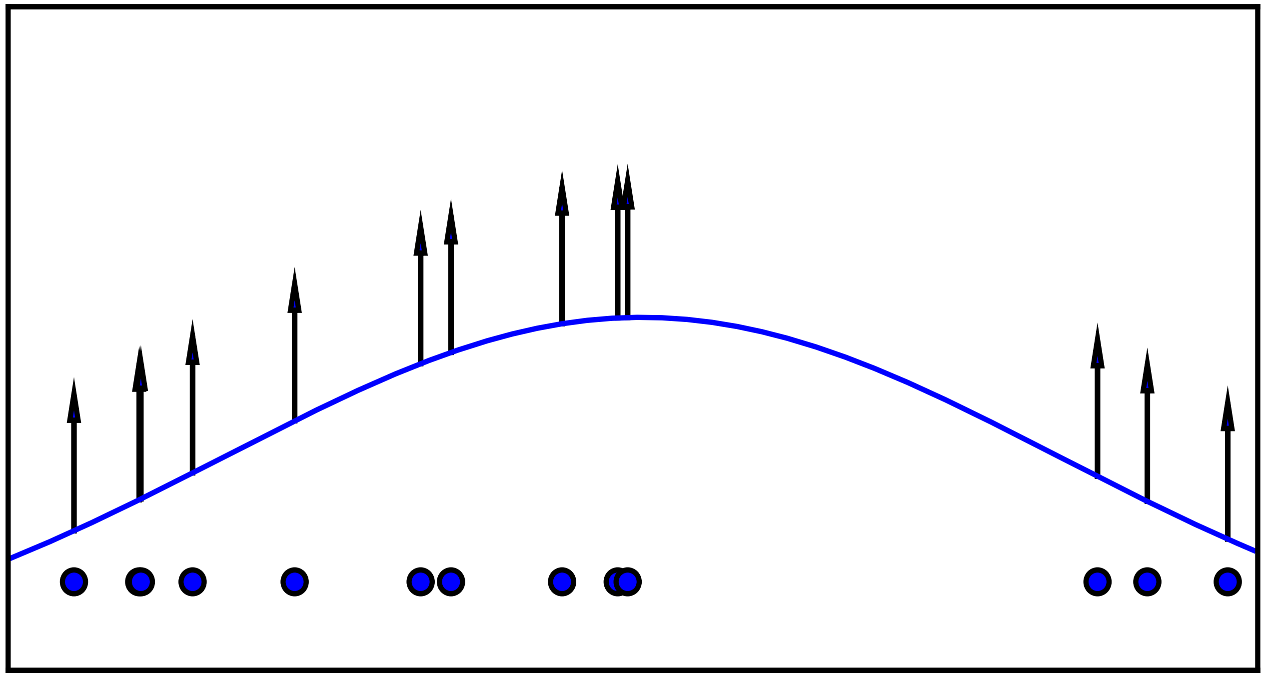
$$\theta^* = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta, c)$$

explicit density

extra conditioning information

Maximum Conditional Likelihood

$$\mathrm{D_{KL}}(P \| Q) = -\sum_{x \in X} P(x) \log\left(\frac{Q(x)}{P(x)}\right)$$

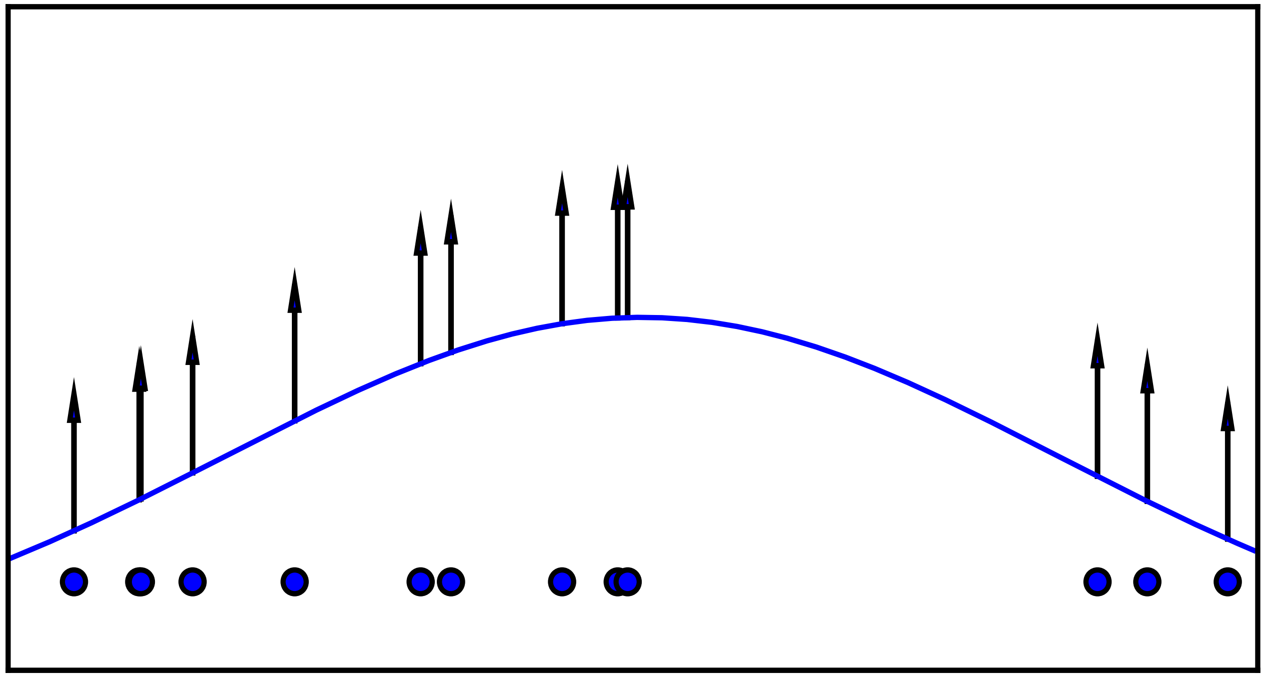


$$\theta^* = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta, c)$$

equiv. to

$$\theta^* = \arg\min_{\theta} \mathrm{D_{KL}}\left(p_{\text{data}} \| p_{\text{model}}(\mathbf{x} \mid \theta, c)\right)$$

Maximum Likelihood-Gaussian with fixed covariance

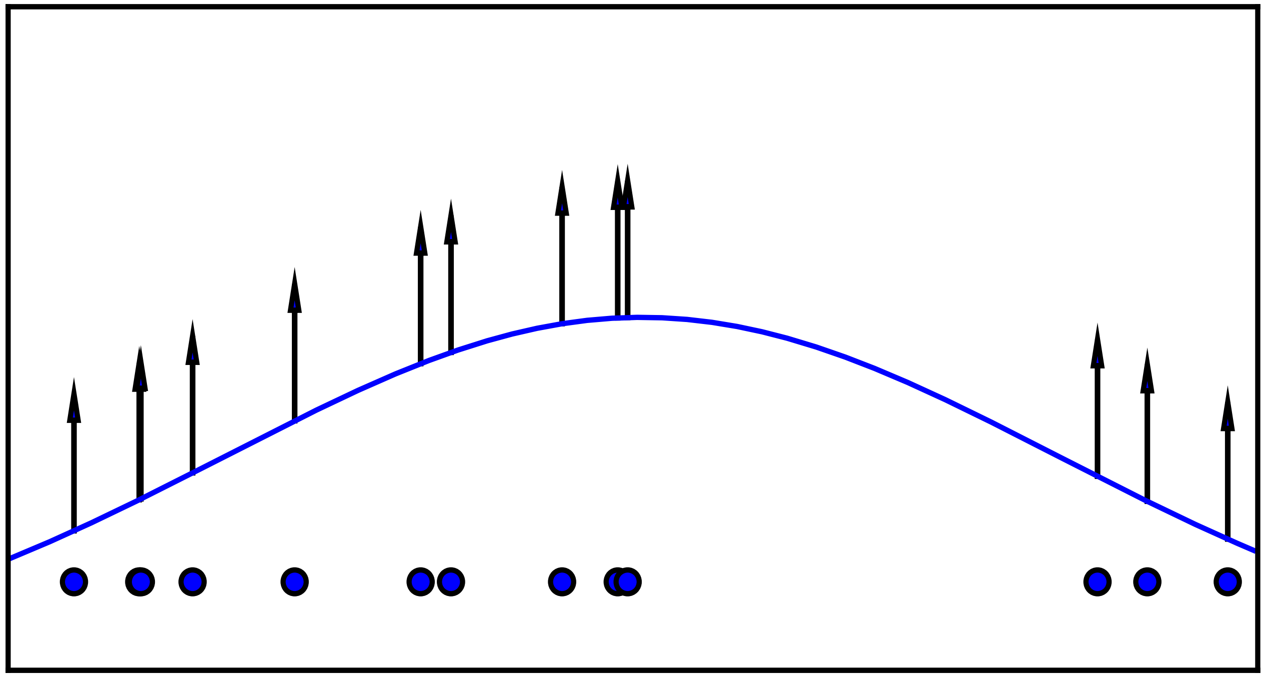


$$\theta^* = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta, c)$$

$$p_{\text{model}}(\mathbf{x} \mid \theta, c) = \frac{1}{(2\pi)^{-\frac{k}{2}} \det(\Sigma)^{-\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu(\theta, \mathbf{c}))^{\top} \Sigma^{-1} (\mathbf{x} - \mu(\theta, \mathbf{c}))\right), \text{ where } \Sigma = \mathrm{I}$$

Maximum Likelihood-Gaussian with fixed covariance

$$p_{\text{model}}(\mathbf{x} \mid \theta, c) = \frac{1}{(2\pi)^{-\frac{k}{2}} \det(\Sigma)^{-\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu(\theta, \mathbf{c}))^\top \Sigma^{-1} (\mathbf{x} - \mu(\theta, \mathbf{c}))\right), \text{ where } \Sigma = \mathrm{I}$$



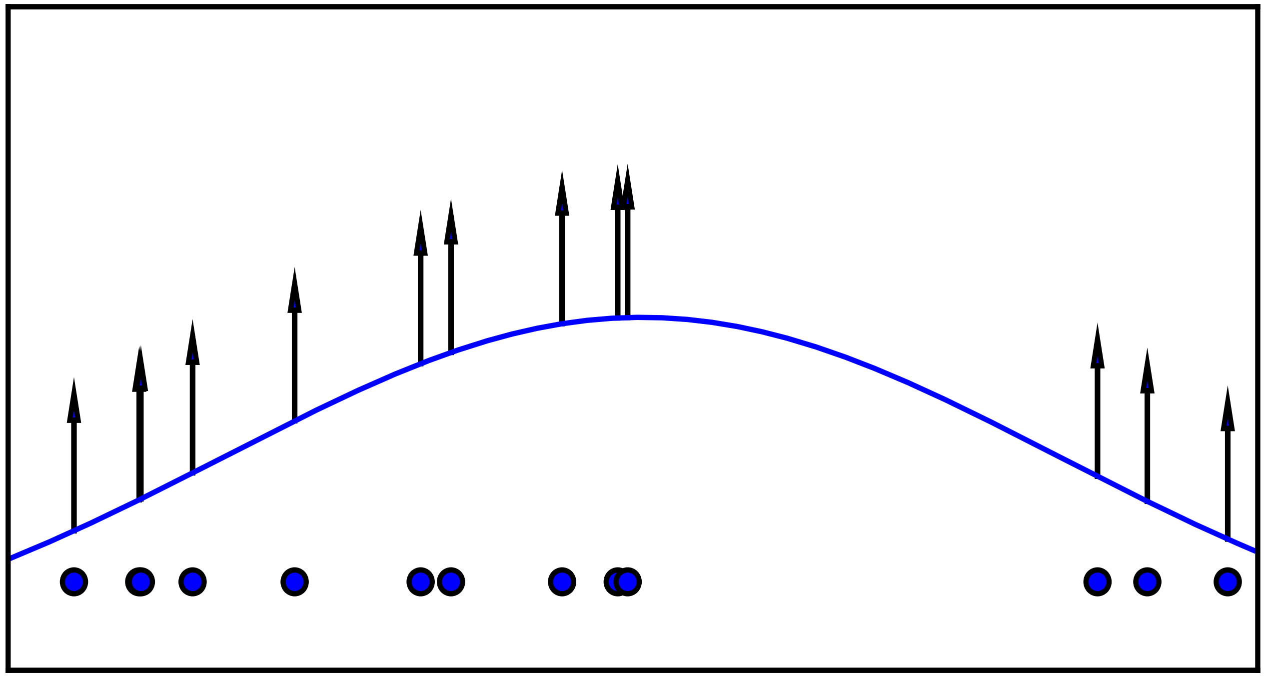
$$\theta^* = \arg\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta, c)$$

$\max_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\mathbf{x} \mid \theta, c)$ equiv. to $\min_{\theta} \mathbb{E}_{x \sim p_{\text{data}}} \|\mathbf{x} - \mu(\theta, \mathbf{c})\|_2^2$

e.g. behavior cloning with continuous actions

$$\mathscr{L}_{BC}(\theta, \mathscr{T}) = \mathbb{E}_{(s_t^j, a_t^j) \sim \mathscr{T}} \left[\|a_t^j - \pi_\theta(s_t^j)\|_2^2 \right]$$

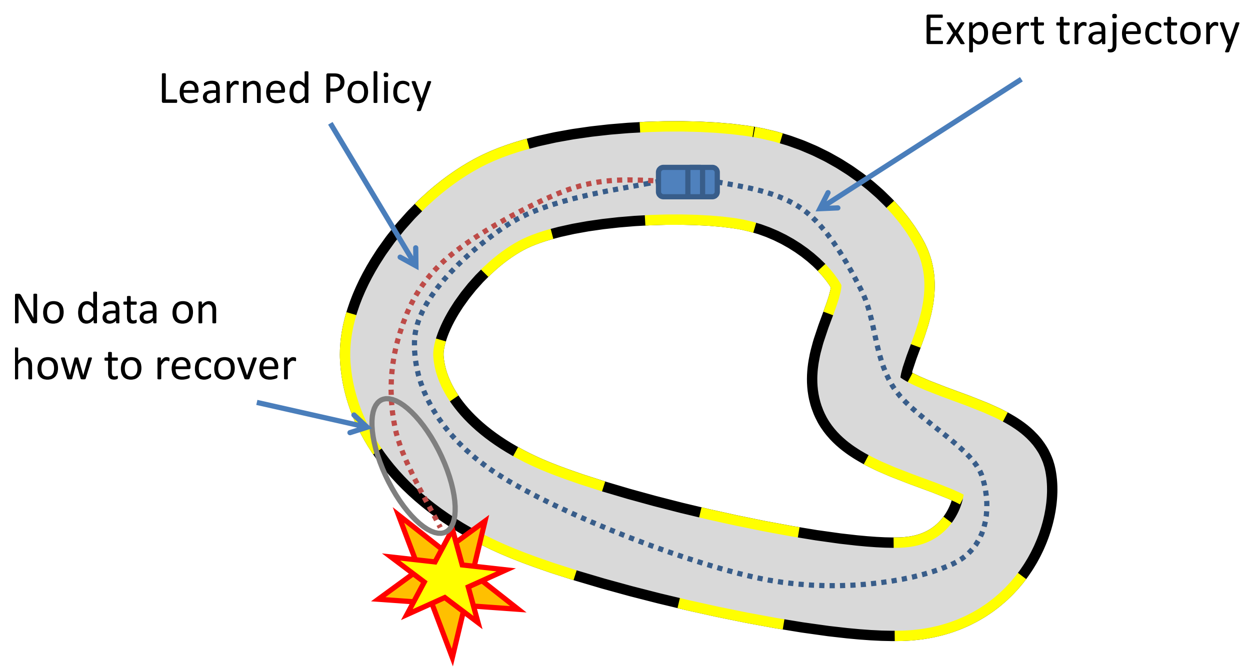
BC Maximizes Conditional Likelihood



$$\mathscr{L}_{BC}(\theta, \mathscr{T}) = \mathbb{E}_{(s_t^j, a_t^j) \sim \mathscr{T}} \left[\|a_t^j - \pi_\theta(s_t^j)\|_2^2 \right]$$

BC Maximizes Conditional Likelihood

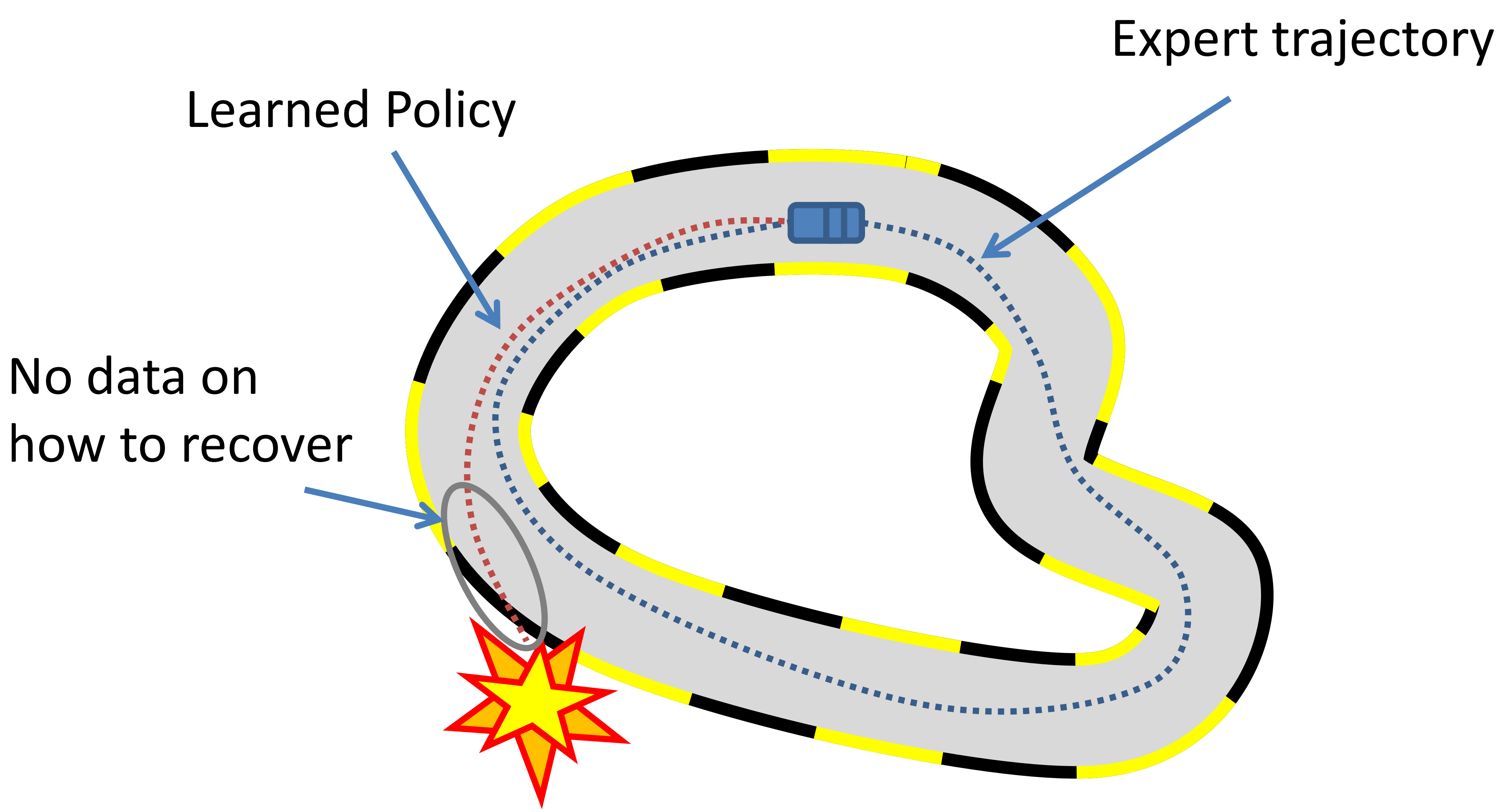
$$\mathscr{L}_{BC}(\theta, \mathscr{T}) = \mathbb{E}_{(s_t^j, a_t^j) \sim \mathscr{T}} \left[\| a_t^j - \pi_\theta(s_t^j) \|_2^2 \right]$$



- Makes the expert actions most likely in the states of the expert trajectories.
- But what about the states not on the expert trajectories? There the actions are unconstrained!

Distribution mismatch (distribution shift)

$$P_{\pi^*}(\mathbf{o}_t) \neq P_{\pi_\theta}(\mathbf{o}_t)$$

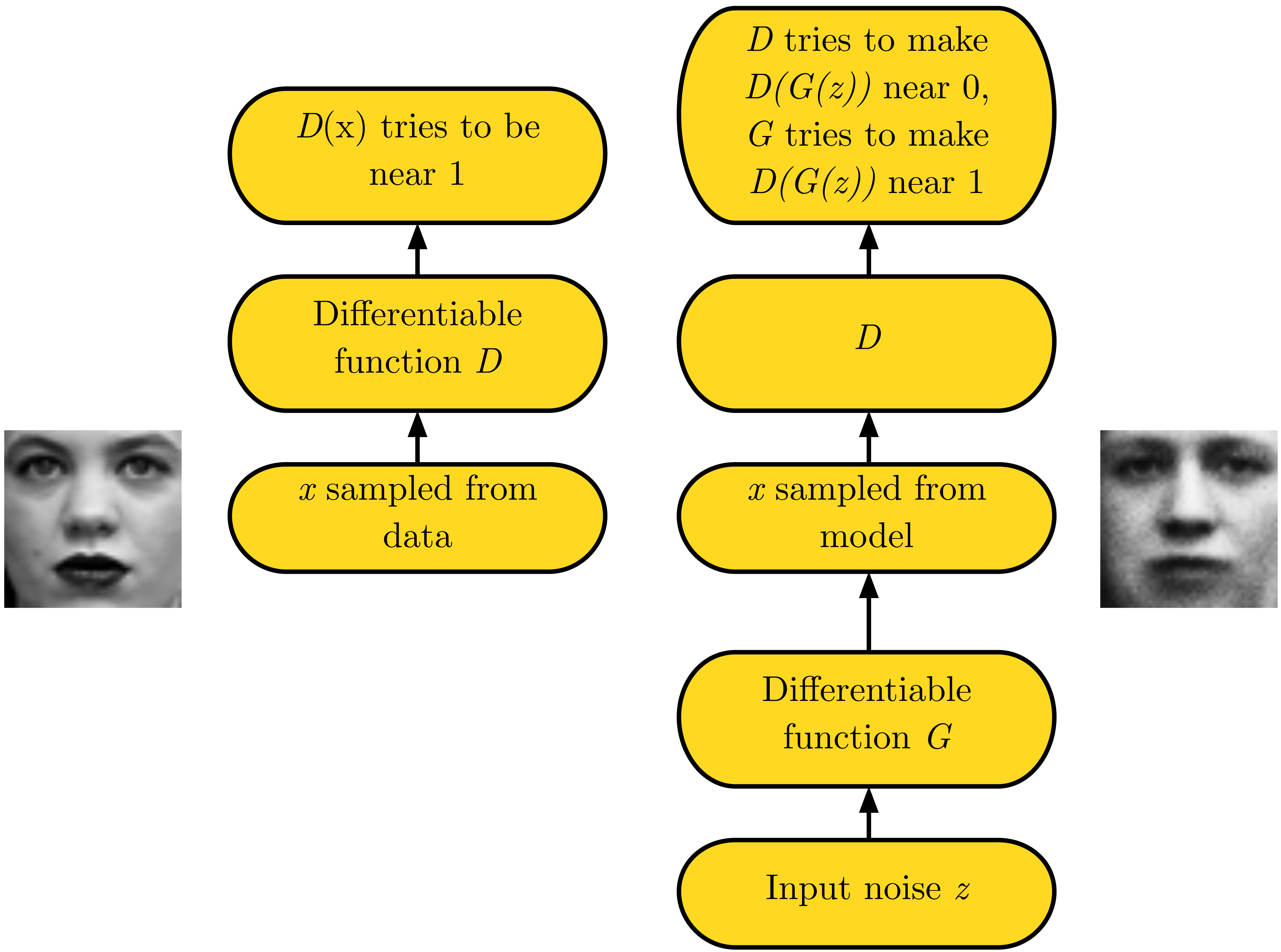


State-action distribution matching objective

- The state-action distribution from the expert trajectories and the state-action distribution that the agent visits by deploying the policy in the environment need to match.

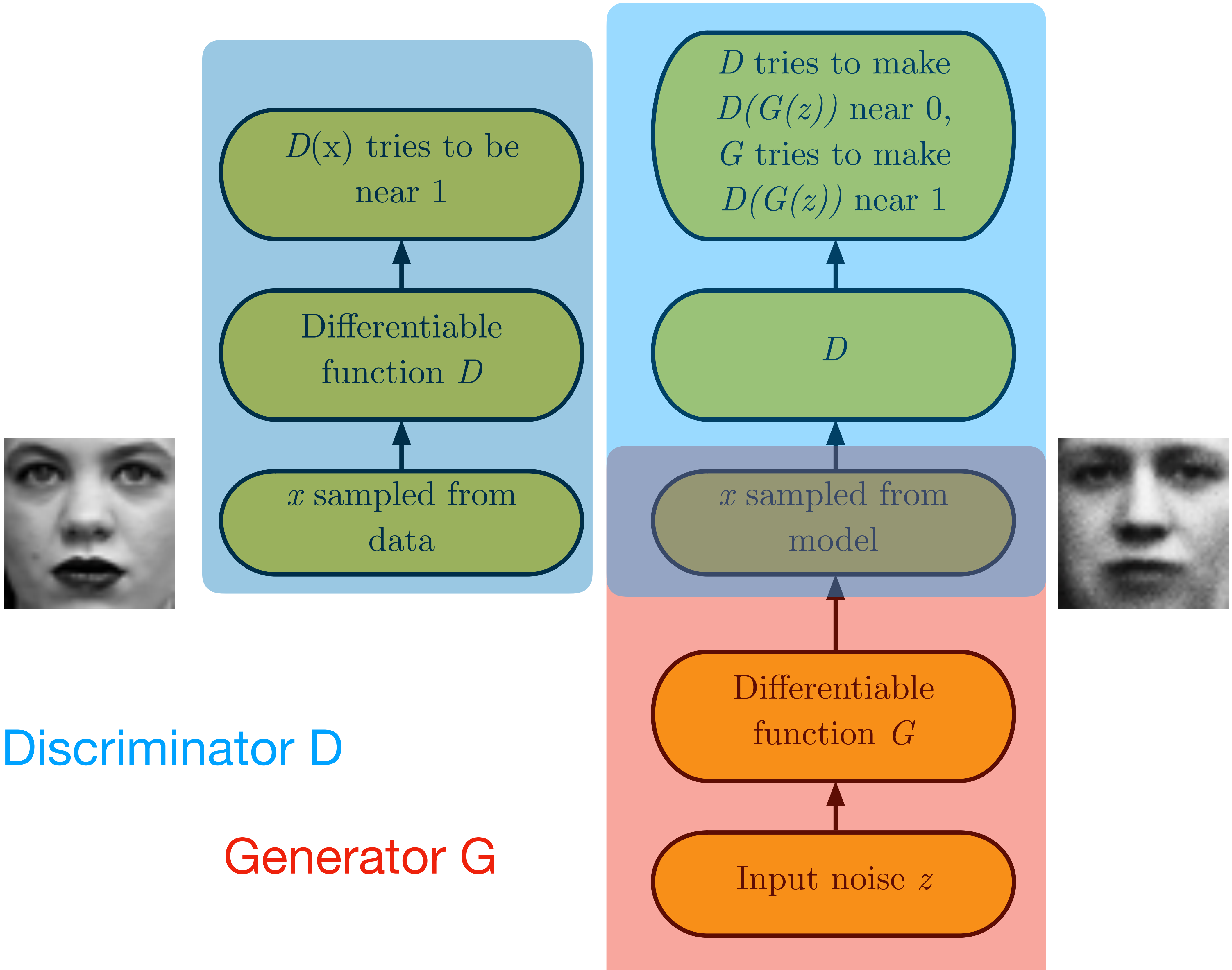
- New solution to the compounding error problem of BC.
- Let’s see how we can optimize this distribution matching objective.

Adversarial Nets Framework



(Goodfellow 2016)

$$\min_{G} \max_{D} V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$



(Goodfellow 2016)

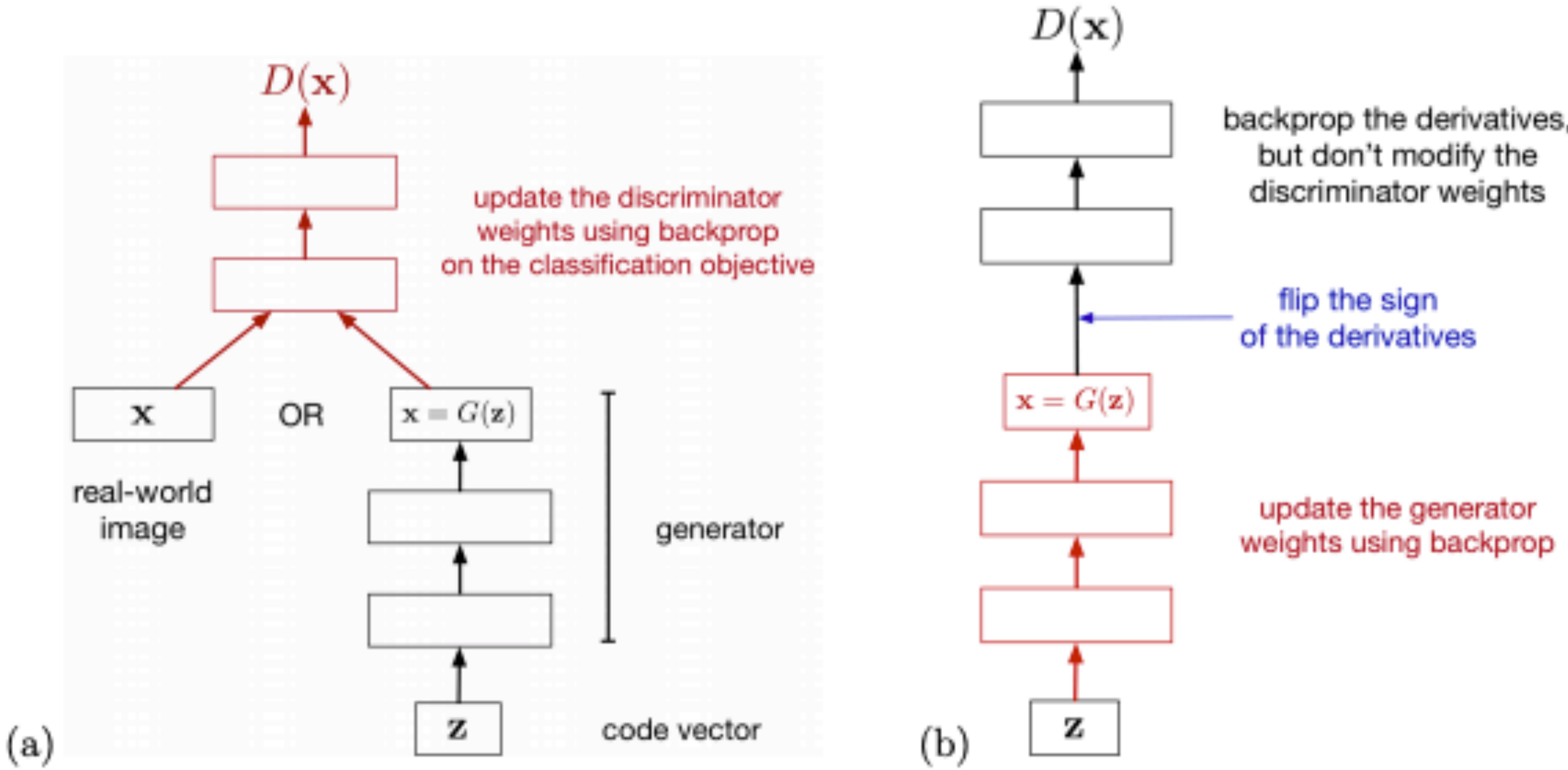
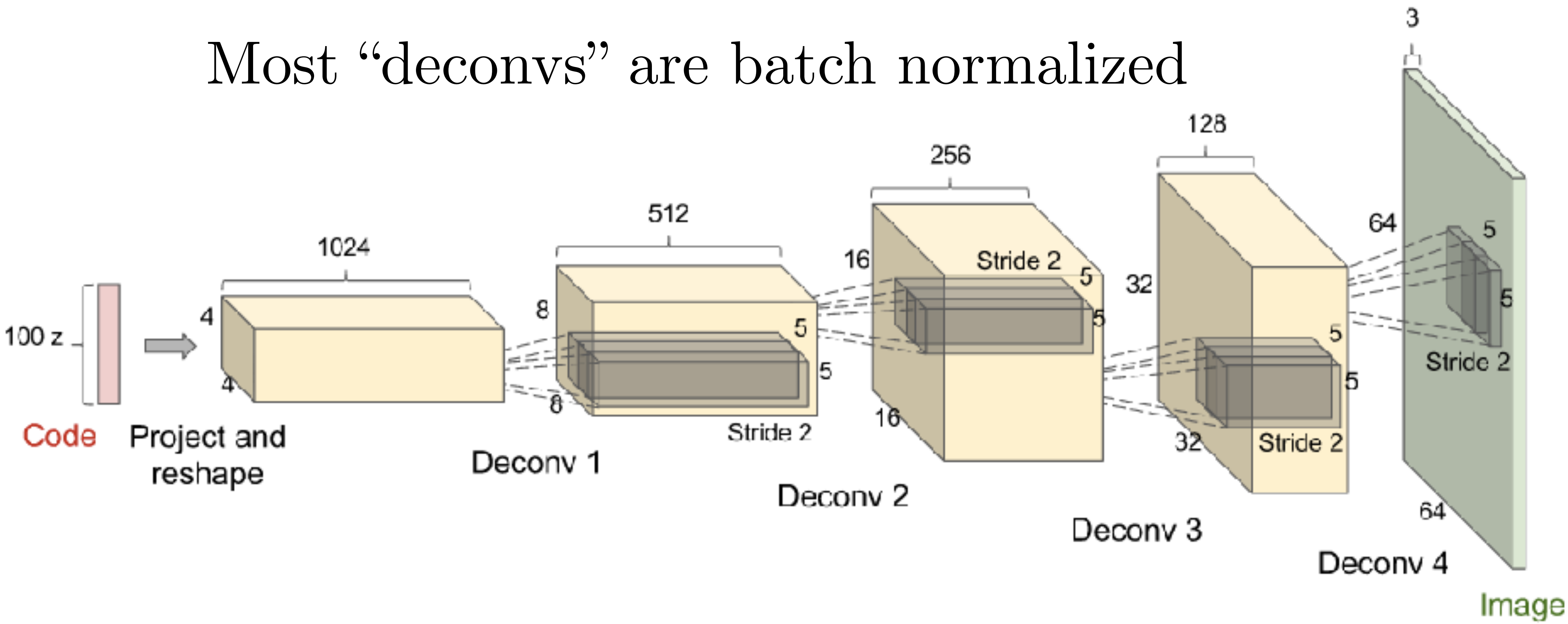


Figure 3: **(a)** Updating the discriminator. **(b)** Updating the generator.

A Generator network (DCGAN)

Most "deconvs" are batch normalized

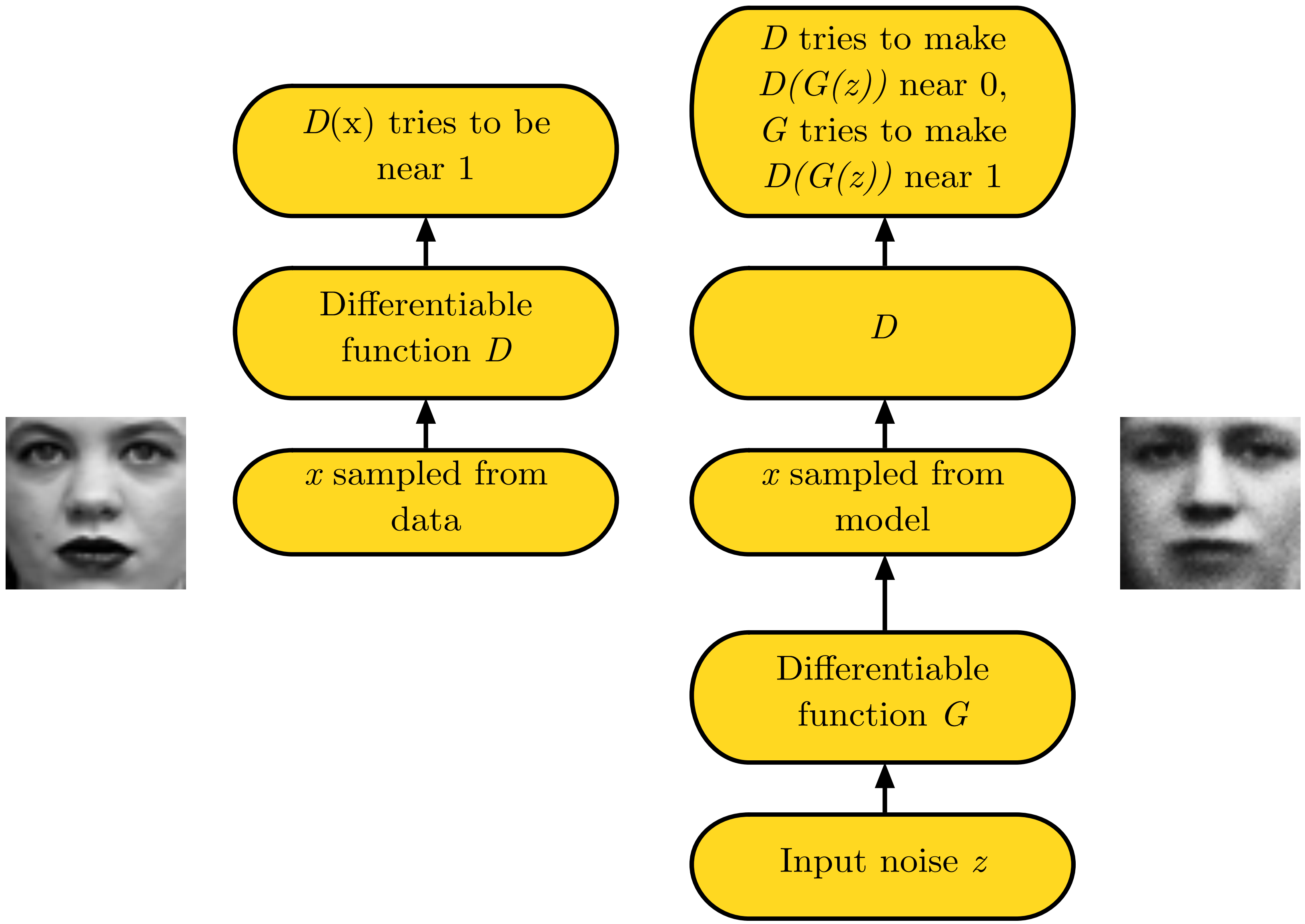


(Radford et al 2015)

(Goodfellow 2016)

Training Procedure

- Use SGD-like algorithm of choice (Adam) on two minibatches simultaneously:
    - A minibatch of training examples
    - A minibatch of generated samples
- Optional: run k steps of one player for every step of the other player.



(Goodfellow 2016)

Questions:

- What if the generator maps all noise vectors to a single super photorealistic image?
- What if we train the discriminator till convergence (it is just a supervised classifier…) and becomes perfect in distinguishing real from generated images?

A minimax game

$$\min_{G} \max_{D} V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

A better cost function

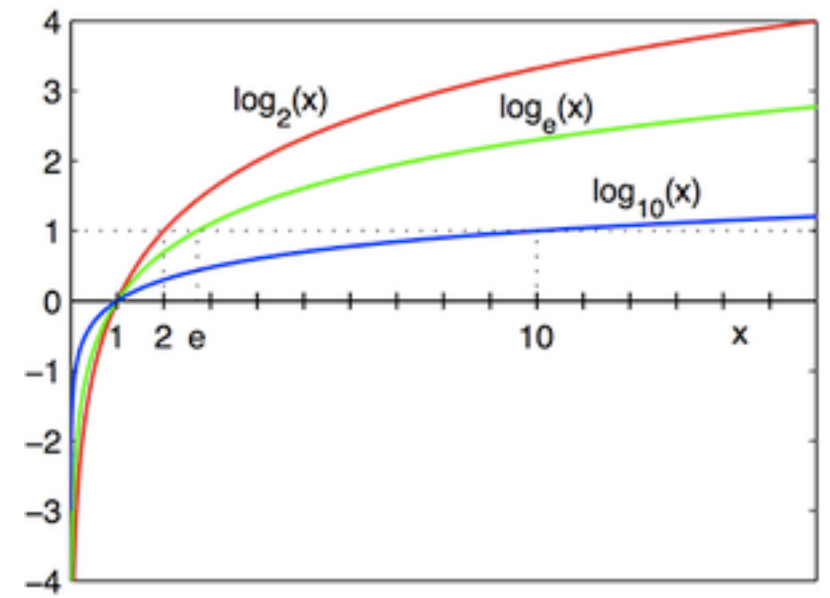
$$\min_G \max_D V(D,G) = \mathbb{E}_{x\sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z\sim p_z(z)}[\log(1-D(G(z)))]$$

$$\min_G \quad \mathbb{E}_{z\sim p_z(z)}[\log(1-D(G(z)))]$$

Gradients not informative when D close to 0

$$\Downarrow$$

$$\min_G \quad \mathbb{E}_{z\sim p_z(z)}[-\log(D(G(z)))]$$



$$\max_D \quad \mathbb{E}_{x\sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z\sim p_z(z)}[\log(1-D(G(z)))]$$

$$\Downarrow$$

$$\min_D \quad \mathbb{E}_{x\sim p_{data}(x)}[\log(1-D(x))] + \mathbb{E}_{z\sim p_z(z)}[\log(D(G(z)))]$$

Optimal discriminator strategy

$$\min_{G} \max_{D} V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

$$V(D, G) = \int_x p_{\text{data}}(x) \log D(x) dx + \int_z p_z(z) \log(1 - D(G(z))) dz$$

Optimal discriminator strategy

$$\min_{G} \max_{D} V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

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$$\int_x p_{\text{data}}(x) \log D(x) dx + \int_x p_G(x) \log(1 - D(x)) dx$$

Optimal discriminator strategy

$$\min_G \max_D V(D,G) = \mathbb{E}_{x\sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z\sim p_z(z)}[\log(1-D(G(z)))]$$

$$\begin{aligned} V(D,G) &= \int_x p_{\text{data}}(x)\log D(x)dx + \int_z p_z(z)\log(1-D(G(z)))dz \\ &= \int_x p_{\text{data}}(x)\log D(x)dx + \int_x p_G(x)\log(1-D(x))dx \\ &= \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1-D(x))dx \end{aligned}$$

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

The discriminator assigns values D(x) to each image x. Let's take the derivative to see where the optimum is attained.

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

$$\frac{d}{dD(x)}\left(p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))\right) = 0$$

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

$$\frac{d}{dD(x)}\left(p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))\right) = 0$$

$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} - p_G(x)\frac{1}{1 - D(x)} = 0$$

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

$$\frac{d}{dD(x)}\left(p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))\right) = 0$$

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$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} = p_G(x)\frac{1}{1 - D(x)}$$

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

$$\frac{d}{dD(x)}\left(p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))\right) = 0$$

$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} - p_G(x)\frac{1}{1 - D(x)} = 0$$

$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} = p_G(x)\frac{1}{1 - D(x)}$$

$$\Leftrightarrow p_{\text{data}}(x)(1 - D(x)) = p_G(x)D(x)$$

Optimal discriminator strategy

$$V(D,G) = \int_x p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))dx$$

$$\frac{d}{dD(x)}\left(p_{\text{data}}(x)\log D(x) + p_G(x)\log(1 - D(x))\right) = 0$$

$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} - p_G(x)\frac{1}{1 - D(x)} = 0$$

$$\Leftrightarrow p_{\text{data}}(x)\frac{1}{D(x)} = p_G(x)\frac{1}{1 - D(x)}$$

$$\Leftrightarrow p_{\text{data}}(x)(1 - D(x)) = p_G(x)D(x)$$

$$\Leftrightarrow D^*(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}$$

Optimal generator strategy

$$C(G) = \max_D V(G, D)$$

Optimal generator strategy

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D^*_G(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D^*_G(G(z))]
\end{aligned}
$$

Optimal generator strategy

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D_G^*(G(z))] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - D_G^*(x)]
\end{aligned}
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Optimal generator strategy

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&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - D_G^*(x)] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)})]
\end{aligned}
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Optimal generator strategy

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&= \mathbb{E}_{x\sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x\sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})]
\end{aligned}
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Optimal generator strategy

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&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - D_G^*(x)] \\
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&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 + \log 4
\end{aligned}
$$

Optimal generator strategy

$$
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C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D_G^*(G(z))] \\
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&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 + \log 4 \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{2p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{2p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4
\end{aligned}
$$

Optimal generator strategy

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D_G^*(G(z))] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - D_G^*(x)] \\
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&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{2p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{2p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] + \mathbb{E}_{x \sim p_G(x)}[\log \frac{p_{\text{G}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] - \log 4
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Optimal generator strategy

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C(G) &= \max_D V(G, D) \\
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&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{2p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{2p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] + \mathbb{E}_{x \sim p_G(x)}[\log \frac{p_{\text{G}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] - \log 4 \\
&= \mathrm{D}_{\mathrm{KL}}\left(p_{data}(x) \,||\, \frac{p_{\text{data}}(x) + p_G(x)}{2}\right) + \mathrm{D}_{\mathrm{KL}}\left(p_G(x) \| \frac{p_{\text{data}}(x) + p_G(x)}{2}\right) - \log 4
\end{aligned}
$$

Optimal generator strategy

$$
\begin{aligned}
C(G) &= \max_{D} V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D_G^*(G(z))] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log D_G^*(x)] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - D_G^*(x)] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(1 - \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)})] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 + \log 4 \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{2p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{2p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] - \log 4 \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] + \mathbb{E}_{x \sim p_G(x)}[\log \frac{p_{\text{G}}(x)}{\frac{p_{\text{data}}(x) + p_G(x)}{2}}] - \log 4 \\
&= \mathrm{D_{KL}}\left(p_{data}(x) \,||\, \frac{p_{\text{data}}(x) + p_G(x)}{2}\right) + \mathrm{D_{KL}}\left(p_G(x) \| \frac{p_{\text{data}}(x) + p_G(x)}{2}\right) - \log 4 \\
&= 2\mathrm{D_{JSD}}\left(p_{data}(x) \,||\, p_G(x)\right) - \log 4
\end{aligned}
$$

Optimal generator strategy

$$
\begin{aligned}
C(G) &= \max_D V(G, D) \\
&= \mathbb{E}_{x \sim p_{data}(x)}[\log \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_G(x)}] + \mathbb{E}_{x \sim p_G(x)}[\log(\frac{p_{\text{G}}(x)}{p_{\text{data}}(x) + p_G(x)})] \\
&= 2\text{D}_{\text{JSD}}\left(p_{data}(x) \,||\, p_G(x)\right) - \log 4
\end{aligned}
$$

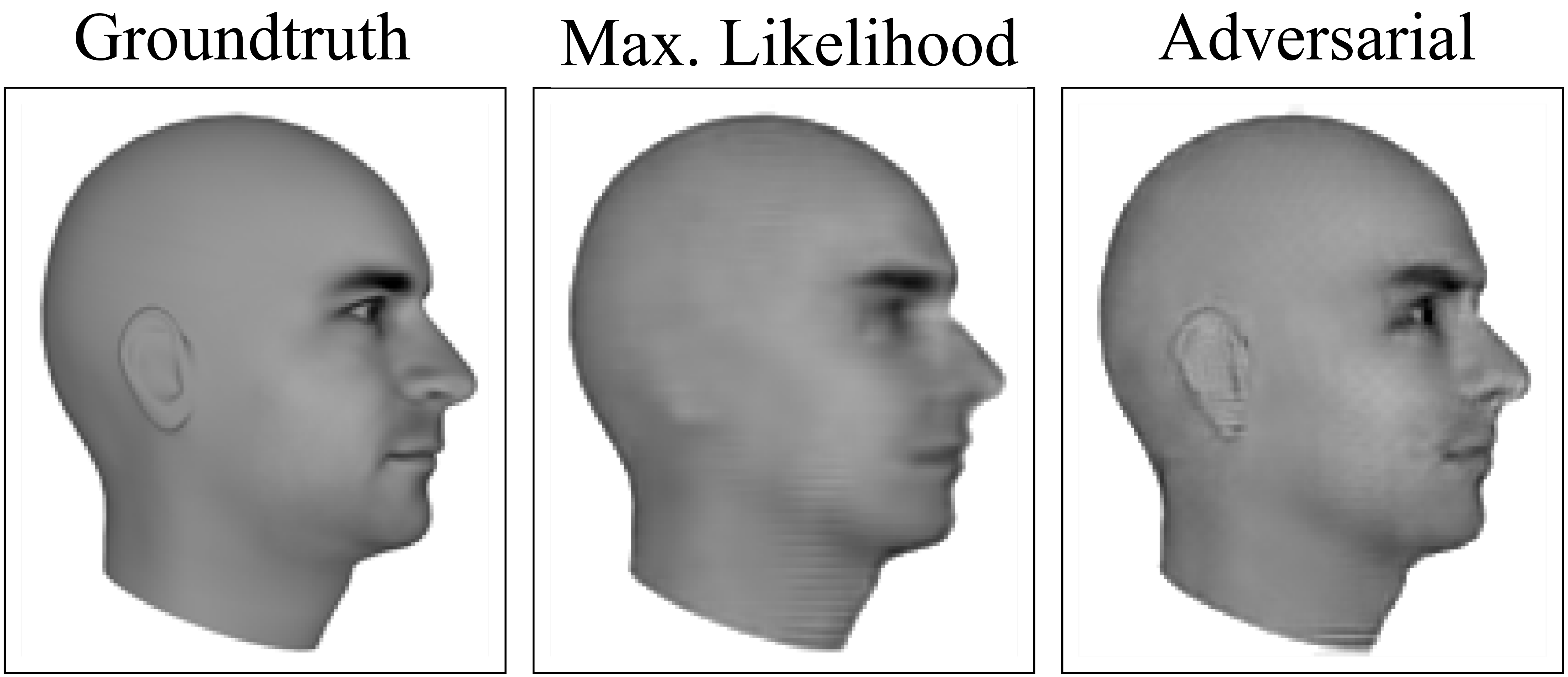
Since $\text{D}_{\text{JSD}} \geq 0, \quad C(G) \geq -\log 4$

By setting $P_G(x) = \text{p}_{\text{data}}(\text{x})$ in the equation above, we get:

$$
C(G) = \mathbb{E}_{x \sim \text{p}_{\text{data}}(x)} \log \frac{1}{2} + \mathbb{E}_{x \sim p_G(x)} \log \frac{1}{2} = -\log 4
$$

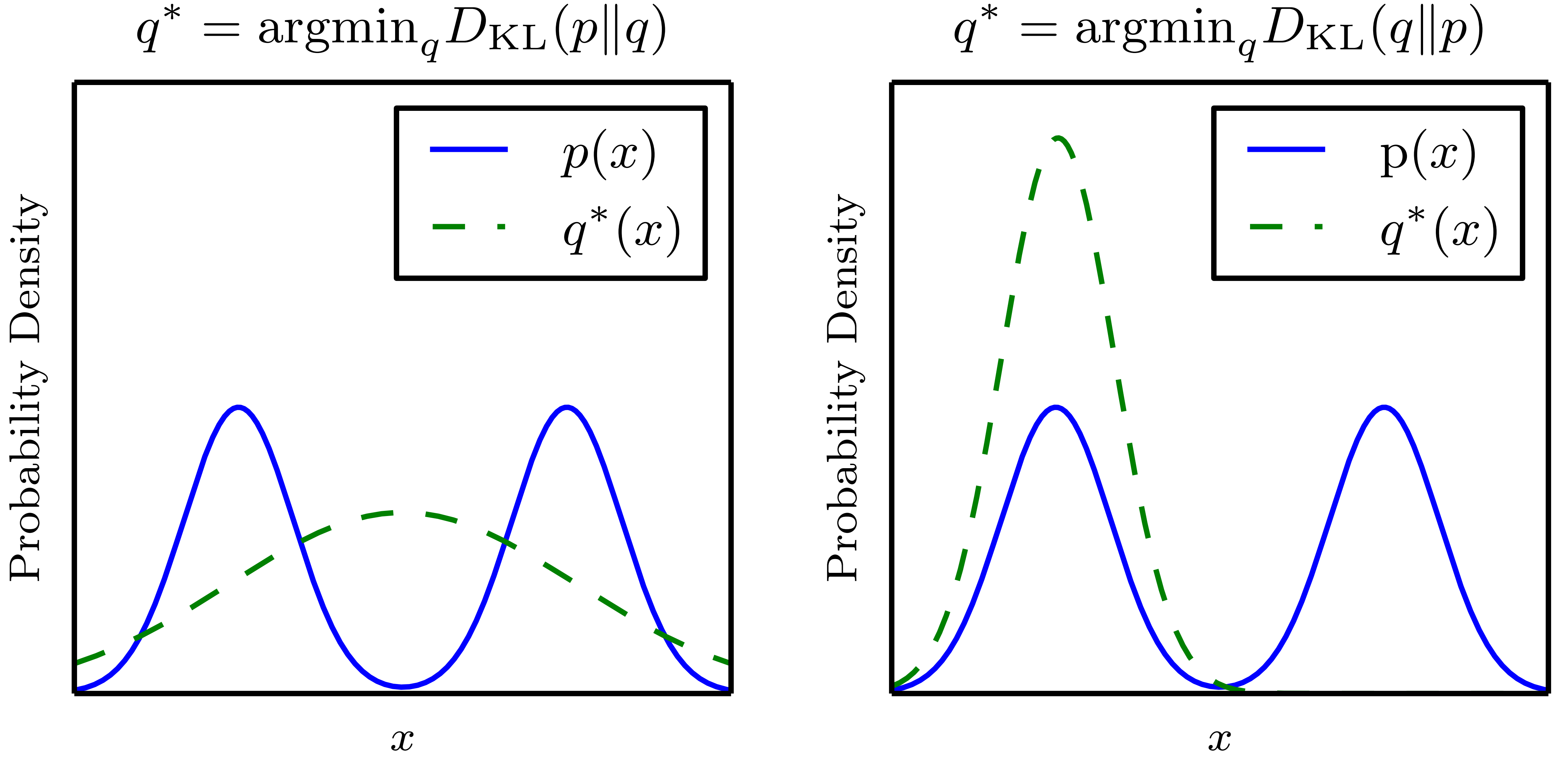
Thus generator achieves the optimum when $P_G(x) = \text{p}_{\text{data}}(\text{x})$.

Next Video Frame Prediction



(Lotter et al 2016)

Maybe an explanation of why GANs work



Maximum likelihood

Reverse KL

Generative Adversarial Imitation learning

The policy network will be our generator, that conditions on the state:

$$\pi_\theta(s) \rightarrow a$$

Generative Adversarial Imitation learning

Find a policy π_θ that makes it impossible for a discriminator network to distinguish between state-action pairs from the expert demonstrations and state-action pairs visited by the agent's policy π_θ:

$$\min_{\pi_\theta} \quad \mathbb{E}_{(s,a)\sim\pi_\theta}[-\log(D_\phi(s,a))]$$

$$\min_{D_\phi} \quad \mathbb{E}_{(s,a)\sim \mathrm{Demo}}[\log(1-D_\phi(s,a))] + \mathbb{E}_{(s,a)\sim\pi_\theta}[\log(D_\phi(s,a))]$$

The reward for the policy optimization is how well I matched the demonstrator's trajectory distribution, else, how well I confused the discriminator.

$$r(s,a) = \log D_\phi(s,a), (s,a) \sim \pi_\theta$$

Generative Adversarial Imitation learning

Input: Expert trajectories , initial policy parameters θ_0 and initial discriminator weights ϕ_0.

For i=0,1,2,3… **do**

1. Sample agent trajectories $\tau_i \sim \pi_{\theta_i}$
2. Update the discriminator parameters with the gradient:

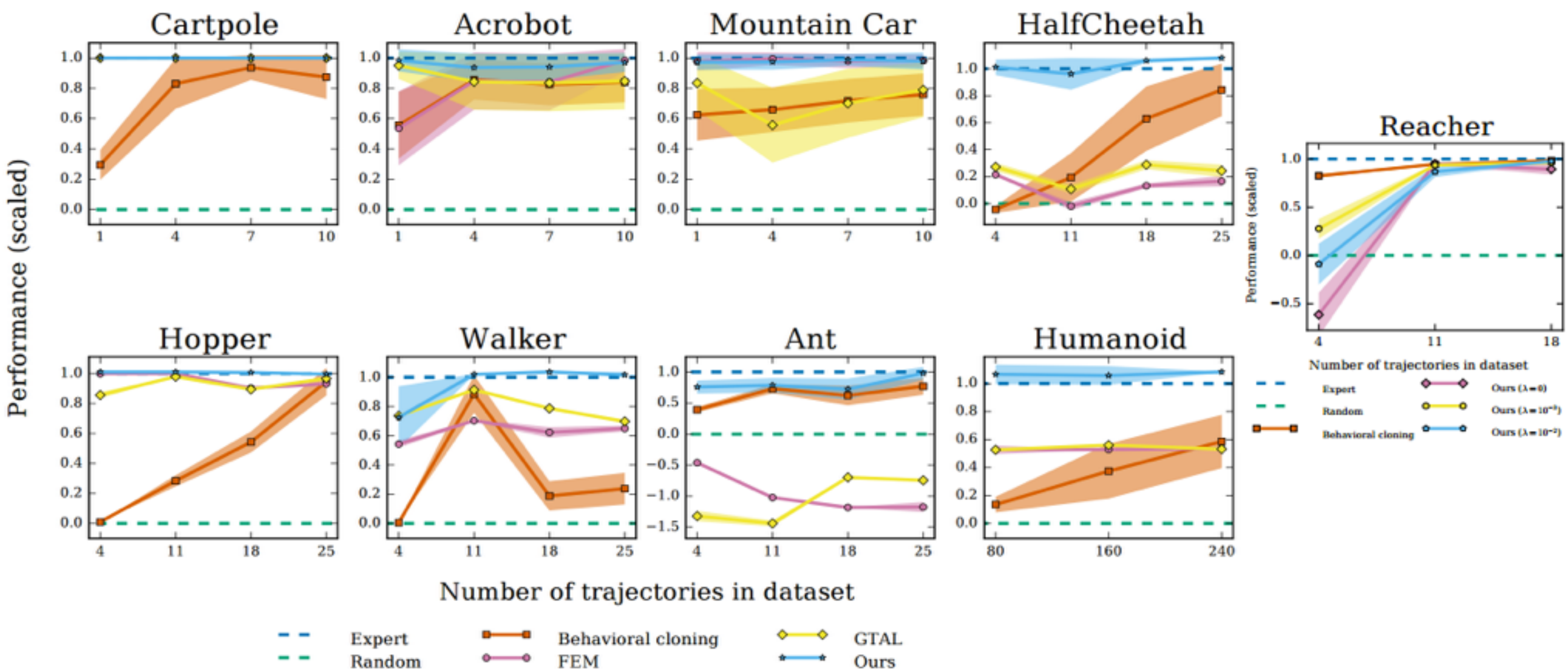
$$\mathbb{E}_{(s,a)\sim \mathrm{Demo}}[\nabla_\phi \log(1 - D_\phi(s,a))] + \mathbb{E}_{(s,a)\in\tau_i}[\nabla_\phi \log(D_\phi(s,a))]$$

3. Update the policy using a policy gradient computed with the rewards, e.g., the REINFORCE policy gradient would be:

$$\mathbb{E}_{(s,a)\in\tau_i}[\nabla_\theta \log \pi_\theta \log D_{\phi_{i+1}}(s,a)]$$

end for

Generative Adversarial Imitation learning



- GAIL: a reinforcement learning method with a reward based on trajectory distribution matching between the agent and an expert.
- BC: reduces imitation learning to supervised learning for individual actions.
- GAIL performs better than behaviour cloning but it requires MORE interactions with the environment.
- Q:Can BC or GAIL outperform the expert?

Imitation learning for diverse goals

- Pushing to diverse locations
- Pouring to different bottles
- Driving to different destinations

We need a way to communicate the goal during learning of the policy

Multi-goal Imitation learning and RL

- Often times we care to learn policies that achieve many related goals
- For example: push object A to (10,10,10) and to (10,12,10)
- The two policies should have many things in common
- Training such policies jointly may be beneficial

Universal value function approximators

$$V(s;\theta) \Rightarrow V(s,g;\theta)$$

$$\pi(s;\theta) \Rightarrow \pi(s,g;\theta) \;\; s,g \in \mathcal{S}$$

- The experience tuples should contain the goal.

$$(s,a,r,s') \Rightarrow \left(s,g,a,r,s'\right)$$

Universal Value Function Approximators, Schaul et al.

Universal value function approximators

$$V(s;\theta) \rightarrow V(s,g;\theta)$$

$$\pi(s;\theta) \rightarrow \pi(s,g;\theta) \quad s,g \in \mathcal{S}$$

What should be my goal representation?

The goal representation is usually the same as your state representation. Usually one of the two:

- **Manual/oracle**: 3d centroids of objects, robot joint angles and velocities, 3d location of the gripper, etc.
- **Learnt**: Some feature encoding over images directly

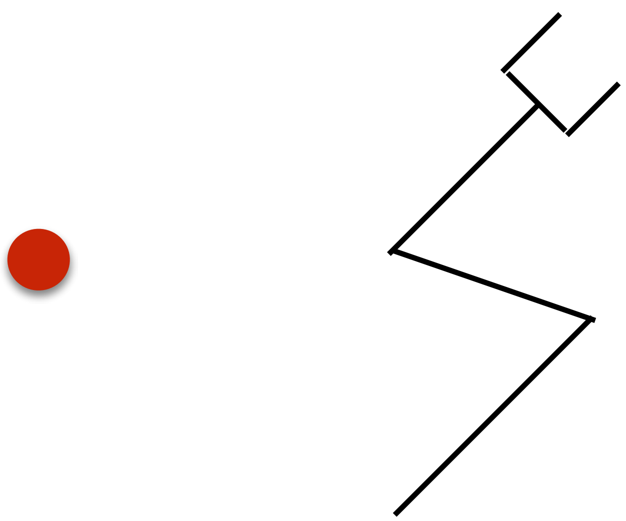
Goal conditioned behavior cloning

- Assumes access to a set of trajectories $\mathcal{T} = \{o_1^j, a_1^j, o_2^j, a_2^j, o_3^j, a_3^j, \ldots, o_T^j, a_T^j, g^j, j = 1 \ldots T\}$. Trains a policy by minimizing a standard supervised learning objective:

$$\mathcal{L}_{BC}(\theta, \mathcal{T}) = \mathbb{E}_{(s_t^j, a_t^j, g^j) \sim \mathcal{T}} \left[\| a_t^j - \pi_\theta(s_t^j, g^j) \|_2^2 \right]$$

Goal relabelling for jointly learning diverse goals

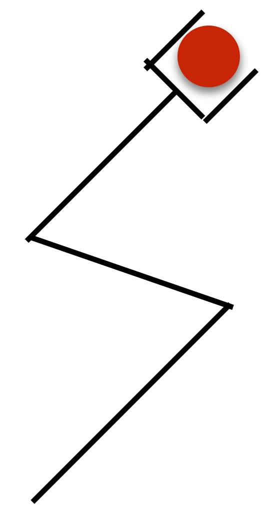
Idea: use failed executions under one goal g, as successful executions under an alternative goal g' (which is where we ended at the end of the episode).



No reward :-(

Goal g

Our reacher at the end of the episode $(s, g, a, 0, s')$



Goal g'

reward :-)

Our reacher at the end of the episode $(s, g', a, 1, s')$

Hindsight Experience Replay

Marcin Andrychowicz[*], Filip Wolski, Alex Ray, Jonas Schneider, Rachel Fong,
Peter Welinder, Bob McGrew, Josh Tobin, Pieter Abbeel[†], Wojciech Zaremba[†]
OpenAI

Idea: use failed executions under one goal g, as successful executions under an alternative goal g' (which is where we ended at the end of the episode).

RL with goal relabelling

Algorithm 1 Hindsight Experience Replay (HER)

Given:

- an off-policy RL algorithm $\mathbb{A}$, ▷ e.g. DQN, DDPG, NAF, SDQN
- a strategy $\mathbb{S}$ for sampling goals for replay, ▷ e.g. $\mathbb{S}(s_0, \ldots, s_T) = m(s_T)$
- a reward function $r : \mathcal{S} \times \mathcal{A} \times \mathcal{G} \to \mathbb{R}$. ▷ e.g. $r(s, a, g) = -[f_g(s) = 0]$

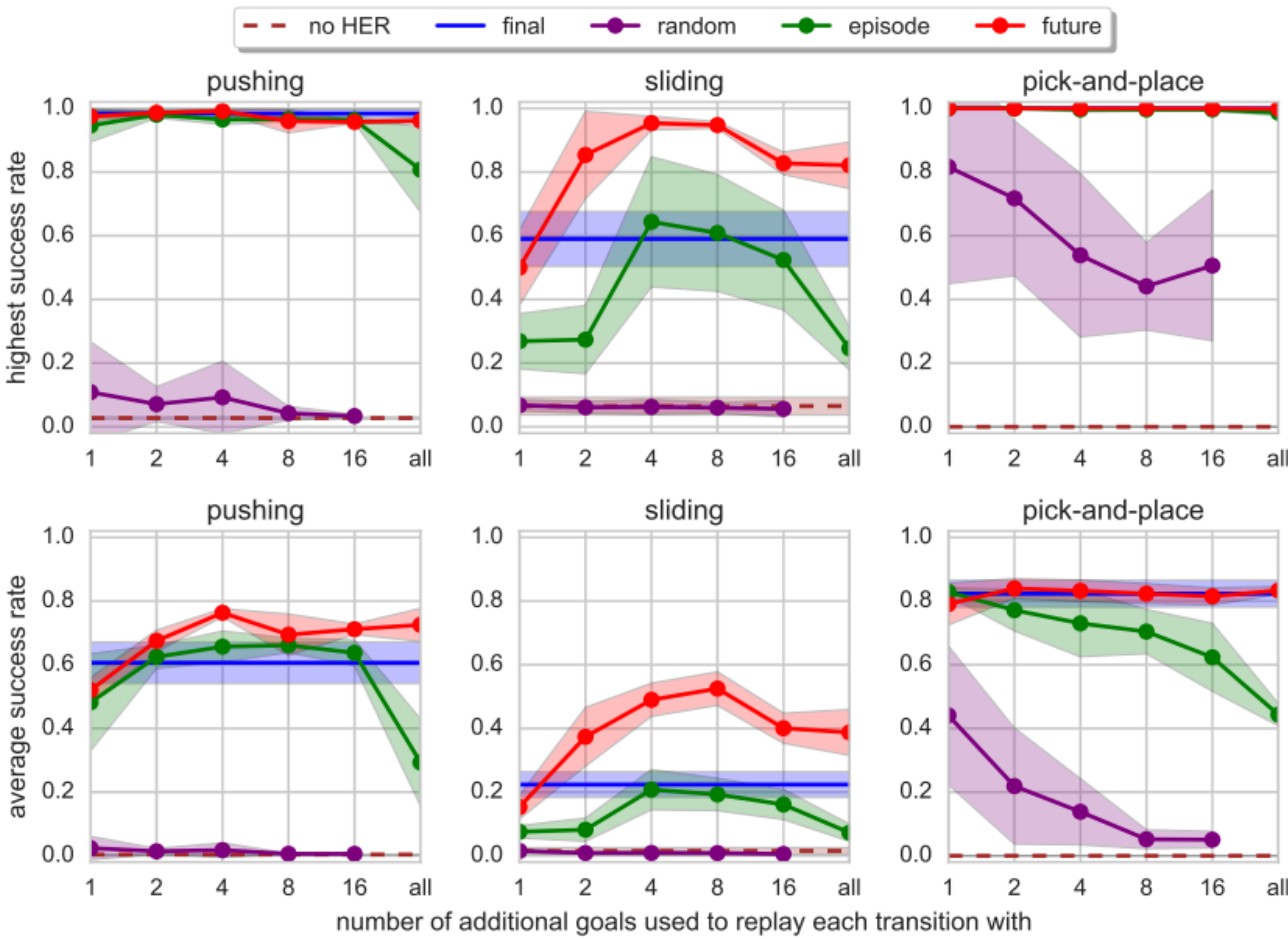
Initialize $\mathbb{A}$ ▷ e.g. initialize neural networks
Initialize replay buffer R
for episode $= 1, M$ **do**
  Sample a goal g and an initial state s_0.
  for $t = 0, T-1$ **do**
    Sample an action a_t using the behavioral policy from $\mathbb{A}$:
      $a_t \leftarrow \pi_b(s_t||g)$ ▷ $||$ denotes concatenation
    Execute the action a_t and observe a new state s_{t+1}
  end for
  for $t = 0, T-1$ **do**
    $r_t := r(s_t, a_t, g)$
    Store the transition $(s_t||g, a_t, r_t, s_{t+1}||g)$ in R ▷ standard experience replay
    Sample a set of additional goals for replay $G := \mathbb{S}(\textbf{current episode})$
    for $g' \in G$ **do**
      $r' := r(s_t, a_t, g')$
      Store the transition $(s_t||g', a_t, r', s_{t+1}||g')$ in R ▷ HER
    end for
  end for
  for $t = 1, N$ **do**
    Sample a minibatch B from the replay buffer R
    Perform one step of optimization using $\mathbb{A}$ and minibatch B
  end for
end for

The reward here is $||s_t - g||$

G : the states of the current episode

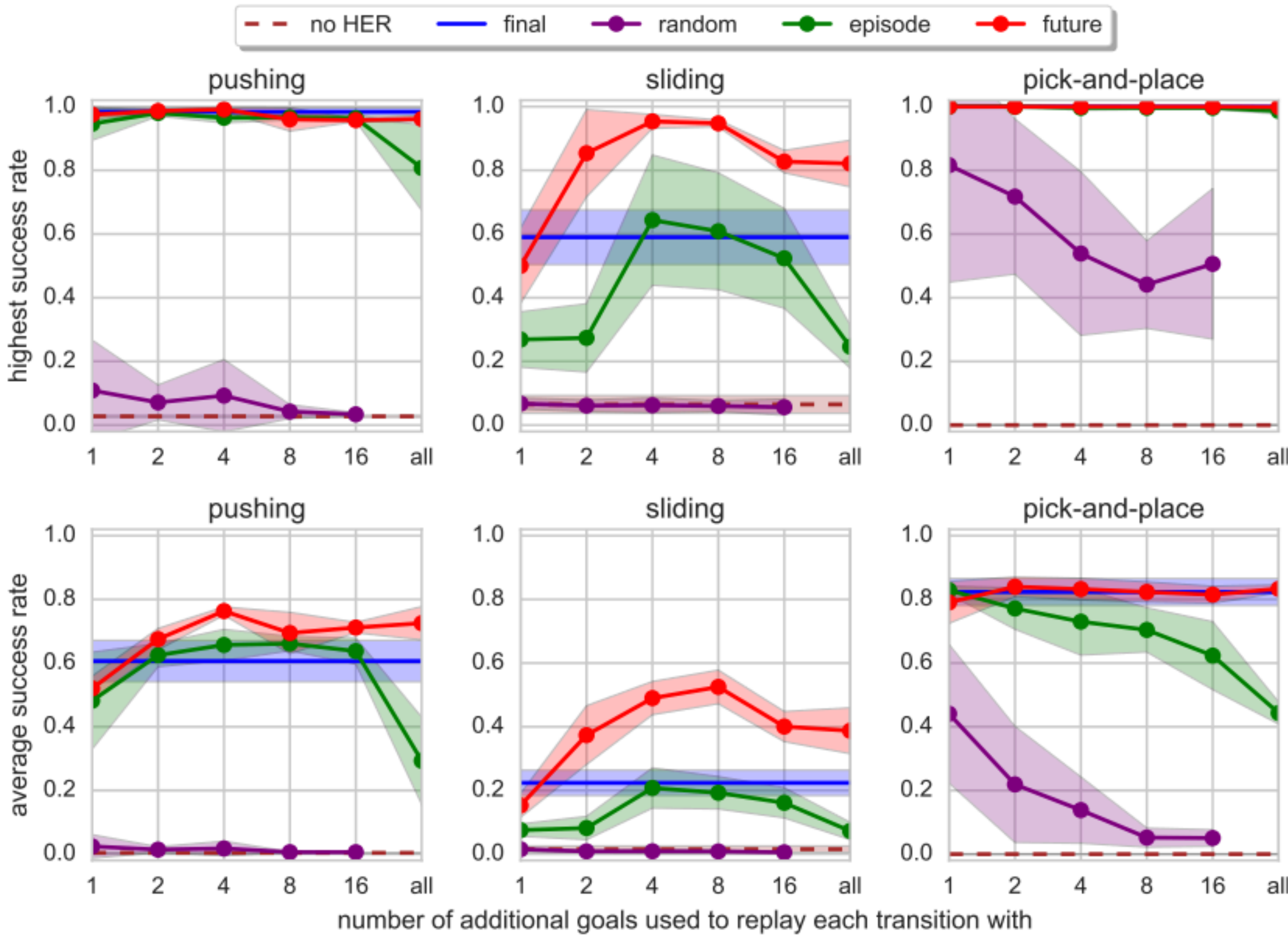
Usually as additional goal we pick the goal that this episode achieved, and the reward becomes non zero

How to select states for relabelling



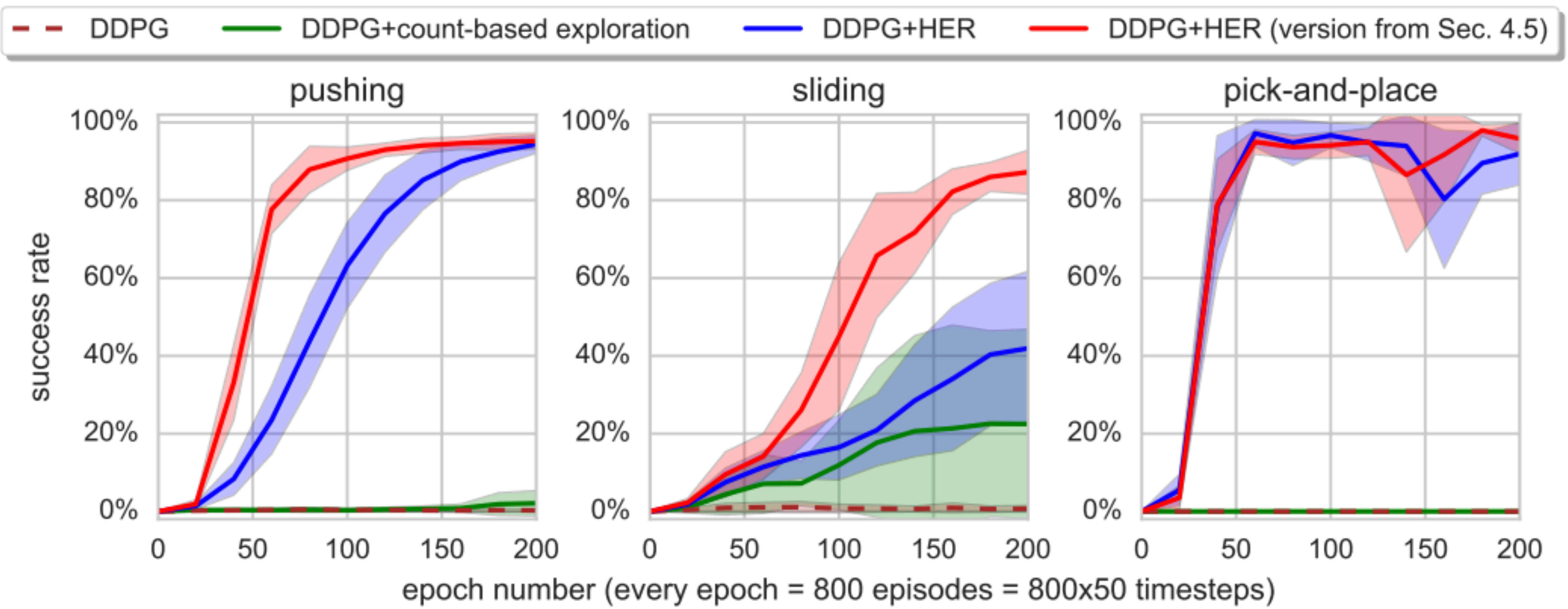
- Final: for each state, use the last state reached in the episode as a goal
- Future: for each state, use random 4 states (observed after the state) in the same episode as a goal

How to select states for relabelling



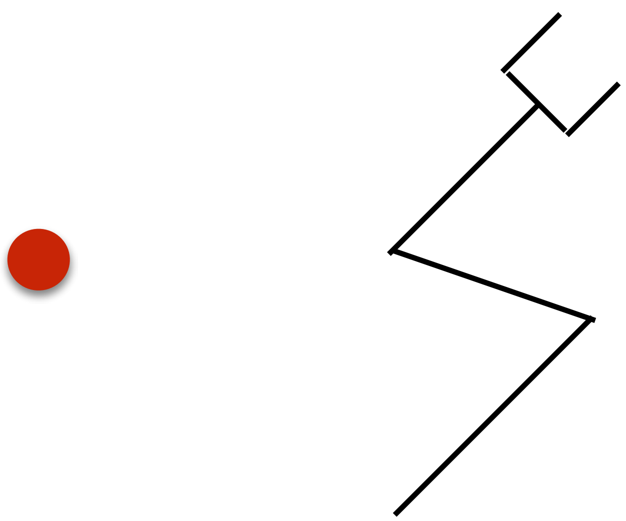
Above 8, the relabelled data are way more than real data, performance degrades.

How to select states for relabelling



Goal relabelling for jointly learning diverse goals

Idea: use failed executions under one goal g, as successful executions under an alternative goal g' (which is where we ended at the end of the episode).

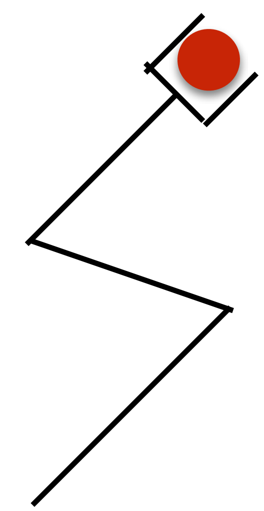


No reward :-(

Goal g

Our reacher at the end of the episode $(s, g, a, 0, s')$

Goal g'



reward :-)

Our reacher at the end of the episode $(s, g', a, 1, s')$

In which of the tasks you think goal relabelling will help the most?

- Picking an object up?
- Reaching?
- Pushing an object to a location?
- All of the above considered jointly?

Goal-conditioned Imitation Learning

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Three ideas:

- Goal-conditioned GAIL
- Combining RL and imitation rewards
- Goal relabelling in both agent and expert trajectories

Goal GAIL

Input: Expert trajectories , initial policy parameters θ_0 and initial discriminator weights ϕ_0.

For i=0,1,2,3… **do**

1. Sample agent trajectories $\tau_i \sim \pi_{\theta_i}$ (for each trajectory, we sample a goal and then run the goal conditioned policy in the environment)
2. Update the discriminator parameters with the gradient:

$$\mathbb{E}_{(s,a,g)\sim \mathrm{Demo}}[\nabla_\phi \log(1 - D_\phi(s,a,g))] + \mathbb{E}_{(s,a,g)\in\tau_i}[\nabla_\phi \log(D_\phi(s,a,g))]$$

3. Update the policy using a policy gradient computed with the rewards, e.g., the REINFORCE policy gradient would be:

$$\mathbb{E}_{(s,a,g)\in\tau_i}[\nabla_\theta \log \pi_\theta \log D_{\phi_{i+1}}(s,a,g)]$$

end for

Combining imitation and task rewards

$$r(s,a) = \lambda r_{GAIL}(s,a) + (1-\lambda) r_{task}(s,a), \quad \lambda \in [0,1]\,.$$

Relabelling expert trajectories

If $(s_t^j, a_t^j, s_{t+1}^j, g^j)$ is in a demonstration trajectory, we also add $(s_t^j, a_t^j, s_{t+1}^j, g' = s_{t+k}^j)$

Data augmentation on demonstrations!

Relabelling expert trajectories

If $(s_t^j, a_t^j, s_{t+1}^j, g^j)$ is in a demonstration trajectory, we also add $(s_t^j, a_t^j, s_{t+1}^j, g' = s_{t+k}^j)$

Data augmentation on demonstrations!

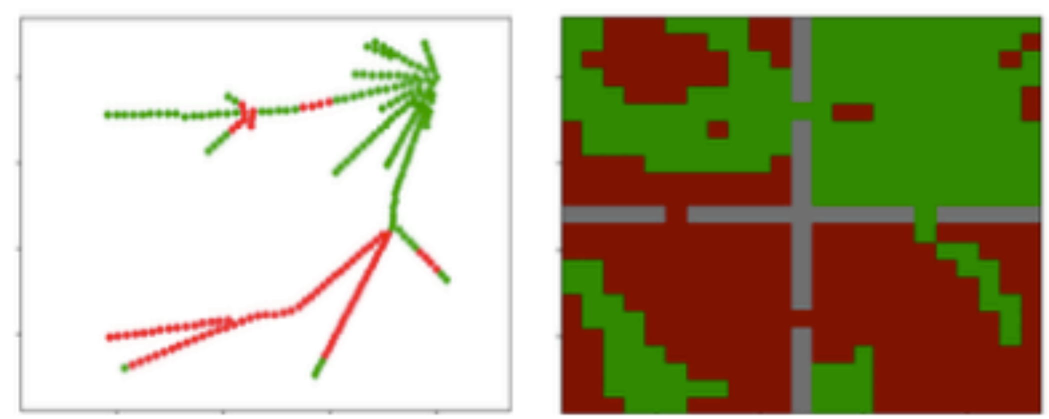
Relabelling can be used in GCBC, GCBC+HER, GoalGAIL,

$$\nabla_\theta \hat{J} = \frac{1}{N} \sum_{i=1}^{N} \nabla_a Q_\phi(a, s, g) \nabla_\theta \pi_\theta(s, g)$$

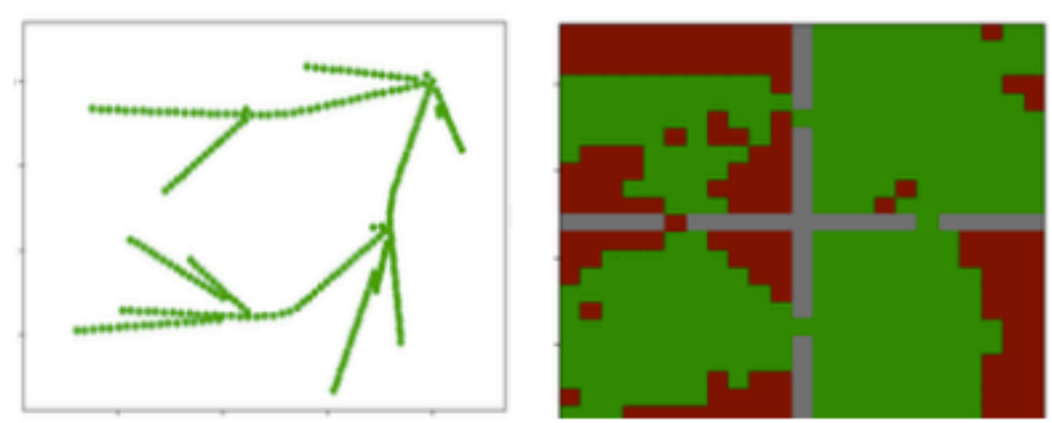
$$\nabla_\theta \mathscr{L}_{BC}(\theta, \mathscr{T}) = \nabla_\theta \mathbb{E}_{(s_t^j, a_t^j, g^j) \sim \mathscr{T}} \left[\|a_t^j - \pi_\theta(s_t^j, g^j)\|_2^2 \right]$$

Green means the policy visited these goals

BC



BC with goal relabelling



Algorithm 1 Goal-conditioned GAIL with Hindsight: *goalGAIL*

Input: Demonstrations $\mathcal{D} = \left\{(s_0^j, a_0^j, s_1^j, ..., g^j)\right\}_{j=0}^{D}$, replay buffer $\mathcal{R} = \{\}$, policy $\pi_\theta(s, g)$, discount γ, hindsight probability p
while not done **do**
  # Sample rollout
  $g \sim \texttt{Uniform}(\mathcal{S})$
  $\mathcal{R} \leftarrow \mathcal{R} \cup (s_0, a_0, s_1, ...)$ sampled using $\pi(\cdot, g)$
  # Sample from expert buffer and replay buffer
  $\left\{(s_t^j, a_t^j, s_{t+1}^j, g^j)\right\} \sim \mathcal{D}$, $\left\{(s_t^i, a_t^i, s_{t+1}^i, g^i)\right\} \sim \mathcal{R}$
  # Relabel agent transitions
  for each i, with probability p **do**
    $g^i \leftarrow s_{t+k}^i, \quad k \sim \mathrm{Unif}\{t+1, \ldots, T^i\}$ ▷ Use *future* HER strategy
  end for
  # Relabel expert transitions
  $g^j \leftarrow s_{t+k'}^j, \quad k' \sim \mathrm{Unif}\{t+1, \ldots, T^j\}$
  $r_t^h = \mathbb{1}\left[s_{t+1}^h == g^h\right]$
  $\psi \leftarrow \min_\psi \mathcal{L}_{GAIL}(D_\psi, \mathcal{D}, \mathcal{R})$ (Eq. $\boxed{3}$)
  $r_t^h = (1 - \delta_{GAIL}) r_t^h + \delta_{GAIL} \log D_\psi(a_t^h, s_t^h, g^h)$ ▷ Add annealed GAIL reward
  # Fit Q_ϕ
  $y_t^h = r_t^h + \gamma Q_\phi(\pi(s_{t+1}^h, g^h), s_{t+1}^h, g^h)$ ▷ Use target networks $Q_{\phi'}$ for stability
  $\phi \leftarrow \min_\phi \sum_h \|Q_\phi(a_t^h, s_t^h, g^h) - y_t^h\|$
  # Update Policy
  $\theta += \alpha \nabla_\theta \hat{J}$ (Eq. $\boxed{2}$)
  Anneal δ_{GAIL} ▷ Ensures outperforming the expert
end while

Goal GAIL without actions

Input: Expert trajectories , initial policy parameters θ_0 and initial discriminator weights ϕ_0.

For i=0,1,2,3... **do**

1. Sample agent trajectories $\tau_i \sim \pi_{\theta_i}$
2. Update the discriminator parameters with the gradient:

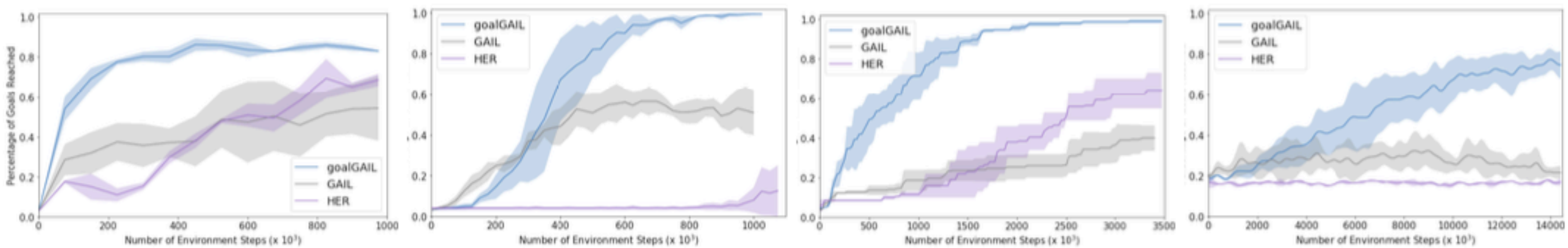
$$\mathbb{E}_{(s,s',g)\sim \text{Demo}}[\nabla_\phi \log(1 - D_\phi(s, s', g))] + \mathbb{E}_{(s,s',g)\in\tau_i}[\nabla_\phi \log(D_\phi(s, s', g))]$$

3. Update the policy using a policy gradient computed with the rewards, e.g., the REINFORCE policy gradient would be:

$$\mathbb{E}_{(s,s',g)\in\tau_i}[\nabla_\theta \log \pi_\theta \log D_{\phi_{i+1}}(s, s', g)]$$

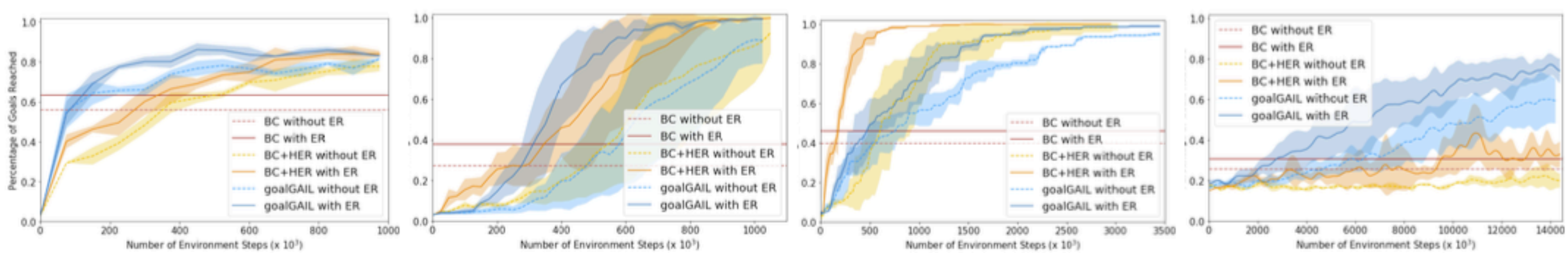
end for

GoalGAIL outperforms GAIL and HER



(a) Continuous Four rooms (b) Pointmass block pusher (c) Fetch Pick & Place (d) Fetch Stack Two

Experience replay helps ALL methods



(a) Continuous Four rooms (b) Pointmass block pusher (c) Fetch Pick & Place (d) Fetch Stack Two

https://sites.google.com/view/goalconditioned-il/

Reinforcement and Imitation Learning for Diverse Visuomotor Skills

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Saran Tunyasuvunakool[‡] János Kramár[‡] Raia Hadsell[‡] Nando de Freitas[‡] Nicolas Heess[‡]
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Ideas:

- Combine imitation and task rewards.
- Start episodes by setting the world in states of the demonstration trajectories.
- Asymmetric actor-critic: the value network takes as input the low-dim state of the system and the policy is trained from pixels.
- Only scene state info to the discriminator
- Co-train the policy CNN with auxiliary tasks
- Sim2REAL via domain randomization.

Combining imitation and task rewards

$$r(s,a) = \lambda r_{GAIL}(s,a) + (1-\lambda) r_{task}(s,a), \quad \lambda \in [0,1]\,.$$

Combining imitation and task rewards

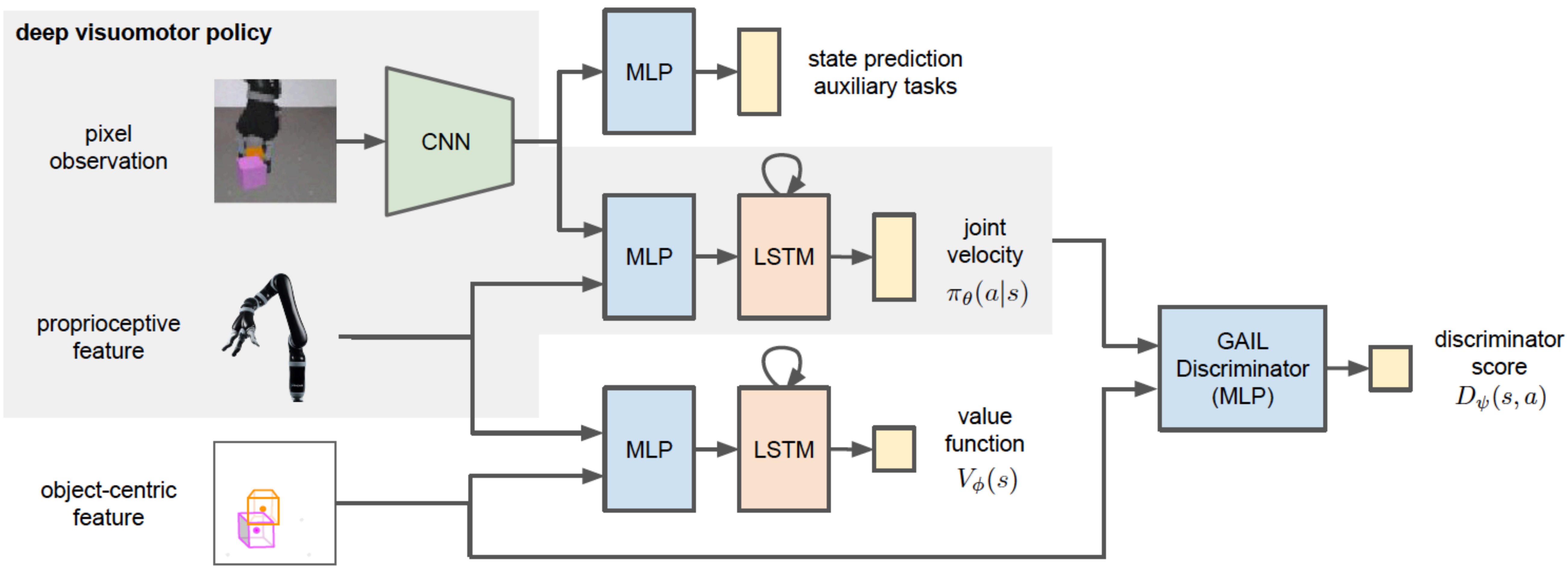
$$\min_{G} \max_{D} V(D, G) = \mathbb{E}_{x \sim p_{data}(x)}[\log D(x)] + \mathbb{E}_{z \sim p_z(z)}[\log(1 - D(G(z)))]$$

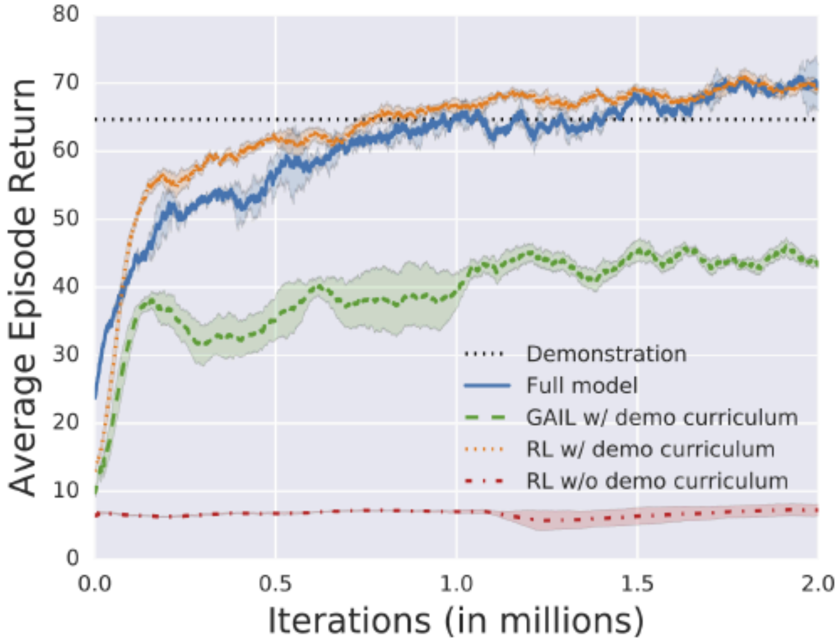
$$r(s, a) = \lambda r_{GAIL}(s, a) + (1 - \lambda) r_{task}(s, a), \quad \lambda \in [0,1] \, .$$

$$r_{GAIL}(s, a) = -\log(1 - D(s, a))$$

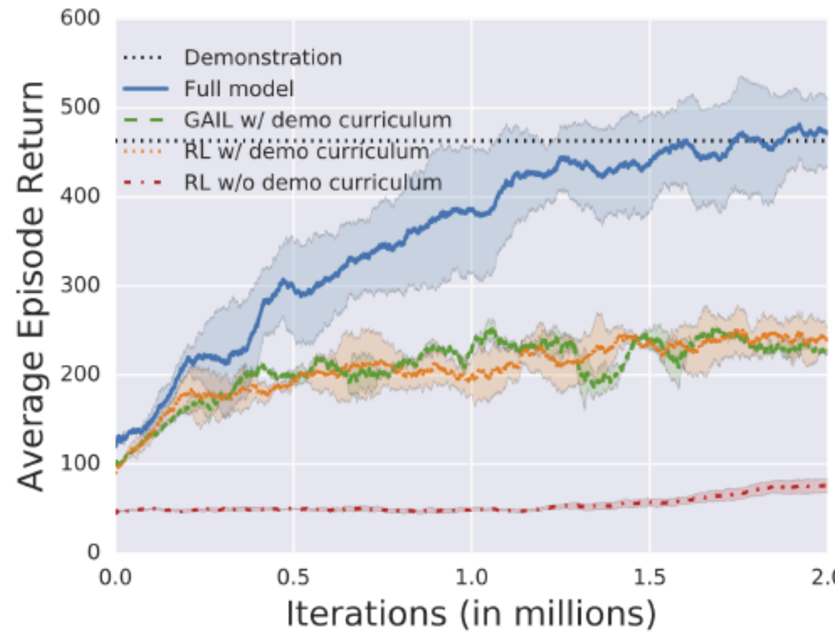
Asymmetric actor-critic

- The value network takes as input the low-dim state of the system (3D object location and velocities and relative distances between objects and the gripper) and the policy is trained from pixels directly. Why?
- This means we need to have access to such state information at training time, but not at test time.

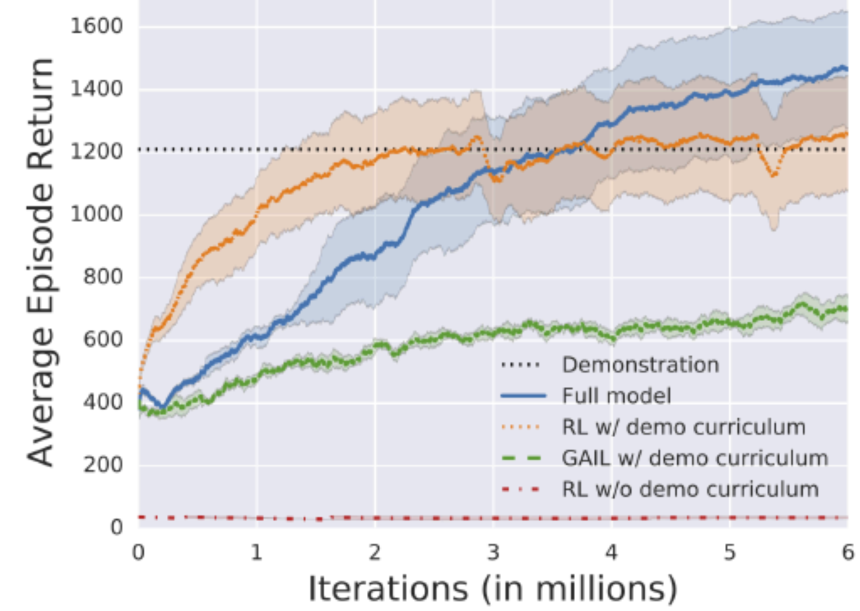

deep visuomotor policy
pixel observation
CNN
MLP
state prediction auxiliary tasks
MLP
LSTM
joint velocity
$\pi_\theta(a|s)$
proprioceptive feature
MLP
LSTM
value function
$V_\phi(s)$
object-centric feature
GAIL Discriminator (MLP)
discriminator score
$D_\psi(s,a)$



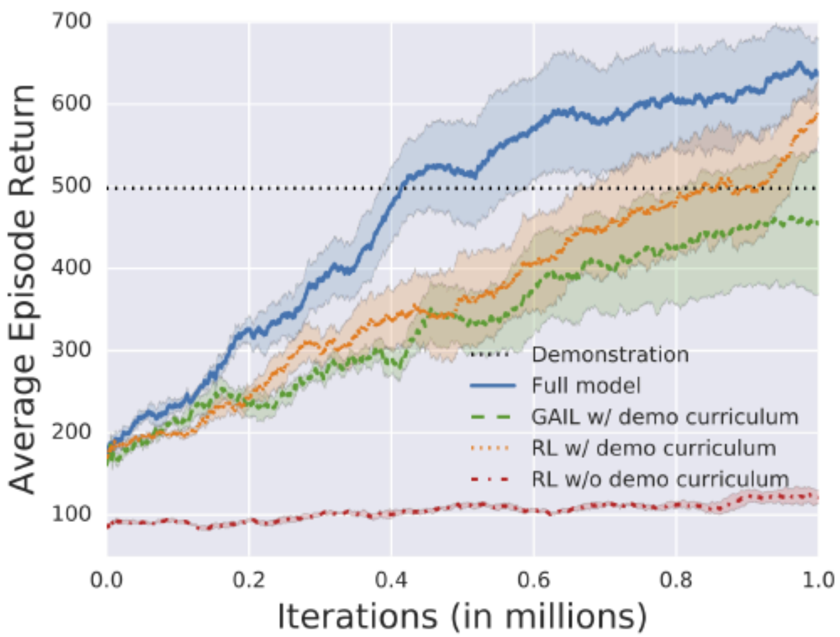
(a) Block lifting



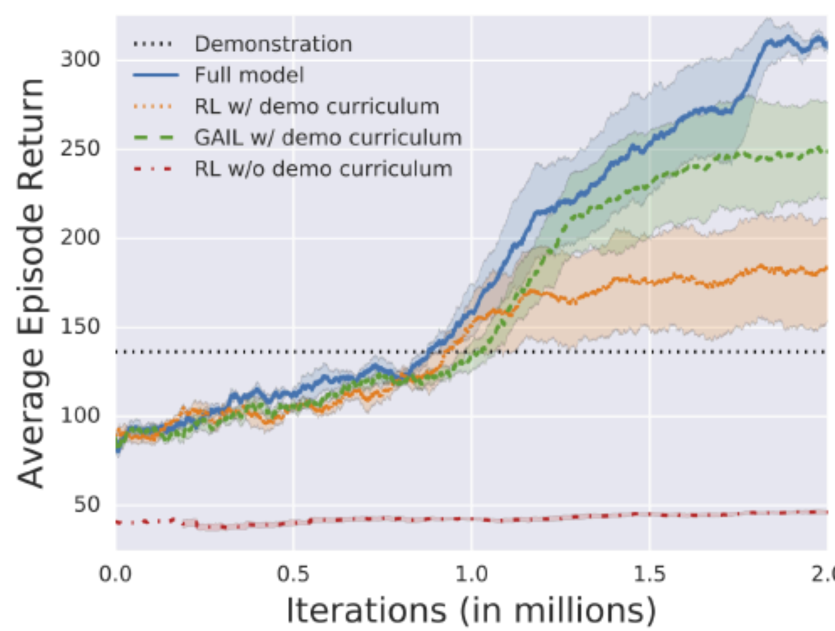
(b) Block stacking



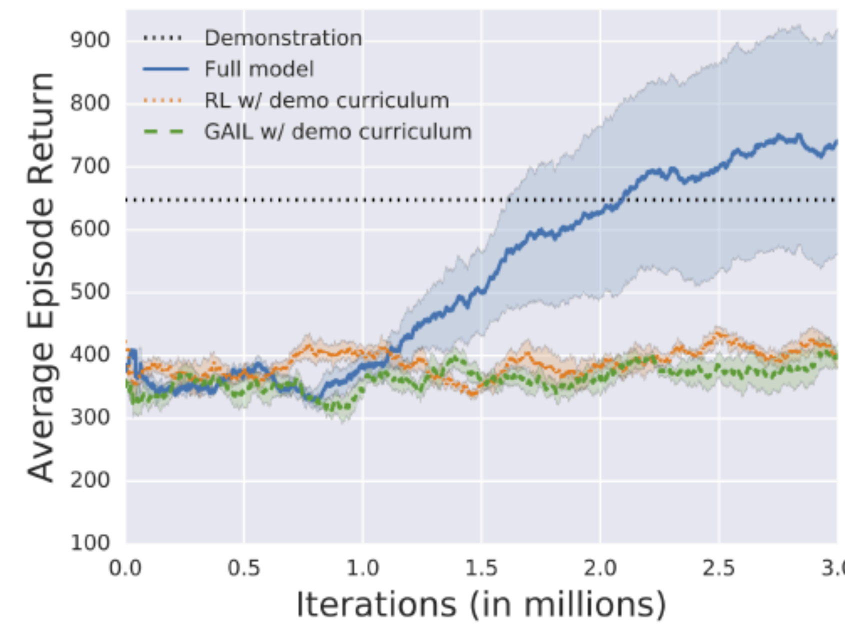
(c) Clearing table with blocks



(d) Clearing table with a box

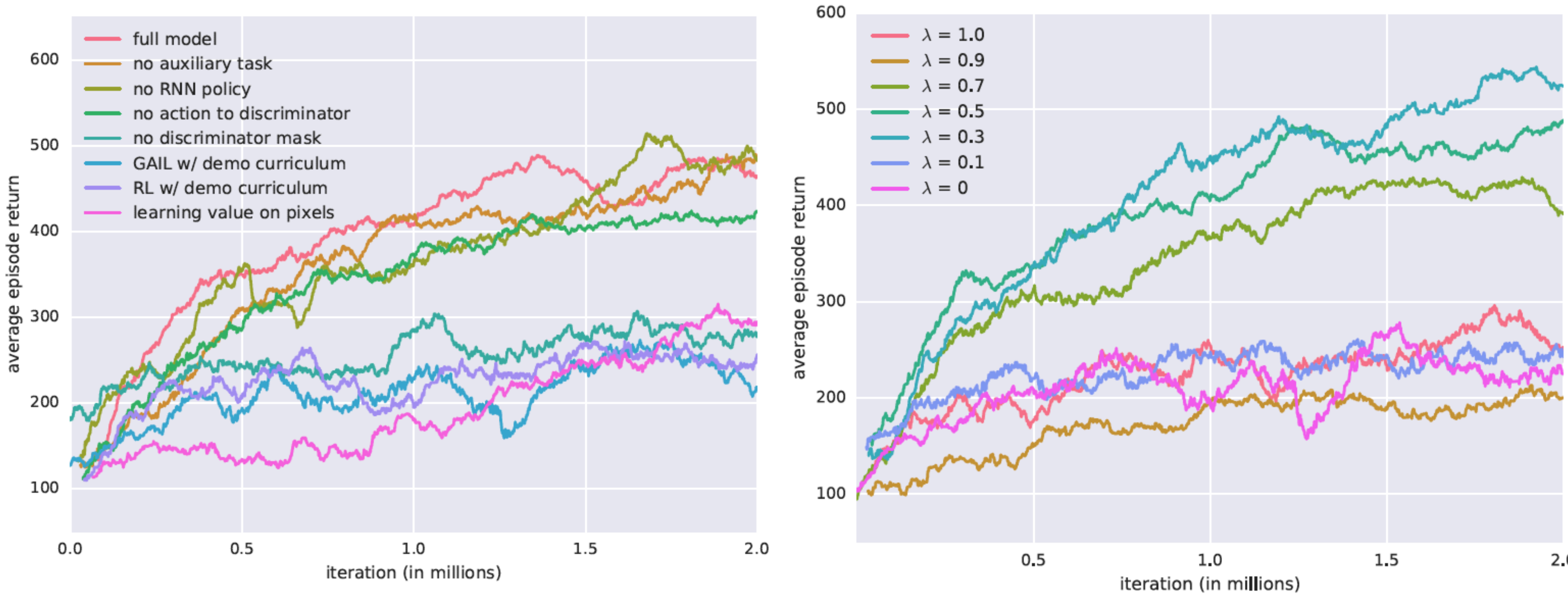


(e) Pouring liquid



(f) Order fulfillment

Resetting on demonstation states is extremely helpful!



(a) Ablation study of model components

(b) Model sensitivity to λ values

- Learning value function from pixels directly is slow
- Not using the GAIL imitation reward but rather using demos just to start episodes in demo states is slow
- No task reward (just imitation) seems not to work. Why?
- No auxiliary task: not big problem.
- Not masking arm info from the discriminator creates problems